Design of Robust Nonlinear Control for Nonlinear Networked Control Systems

Abdul-Wahid A. Saif, Abdallah AL-Shihri, Moustafa Elshafei

Abstract- In this paper, the design of nonlinear observer-based dynamic controller for Nonlinear Networked Control Systems (NNCS) will be presented on the assumption that there exist packet loss between the sensor and the controller and between the controller and the actuator. The existence of the packet loss will be presented by different linear function of a stochastic variable satisfying a Bernoulli random binary distribution. The formulation of NNCS problem will be reduced to solving Linear Matrix Inequality (LMI). Finally, a simulation example will be presented to demonstrate the effectiveness of the proposed LMI approach.

Keywords:Nonlinear Control, Nonlinear Networked Control Systems, Robust Design, LMI, Communication Networks

I. INTRODUCTION

In general, NCS researches concentrate on two major issues either research in the control of the network or in the control over network. The study for the control of networks concentrates on the communications and the networks that will be used for the real time NCSs, e.g., routing control, congestion reduction, efficient data communication, networking protocol, etc. The other type is looking for the quality of the control for the plants through the networks to minimize the negative effects of the networks on the NCS performance such as network delay, packet dropout or disorder of packet arrival. It is clear from all these researches that the goal of them was to find high in QoS (Quality of Service) and high QoC (Quality of Control).This paper is under the QoC category. There is a tremendous literature survey on the stability of linear Networked Control Systems (NCS) to solve the major problems of the NCS which are the time delay, packet dropout and the disorder of receiving packets., see for example the latest references [5,10,11] and the references therein. The research for the Nonlinear Networked Control Systems (NNCS) can be considered as extension for the approaches used to solve these problems in the linear NCS case, but these methods were extended for the NNCS with the suitable assumptions and conditions. In [9], the authors tried to find an analysis of robust stability for uncertain nonlinear networked control systems. So, they startedthe research from modeling the nonlinear systems with induced delay time and considered the uncertainty in the model. Even though they tried to find a stability analysis for the nonlinear NCS with more accuracy but the model was not assumed to face a packet loss and bandwidth limitation.

In [12] the authors provided theoretical results with some assumptions for stabilization of NNCS based on the Model Predictive Control (MPC). The uncertainty was considered there with Networked Delay Compensation (NDC) strategy. The constraints suggested there were to help in predicting the model results. It is proposed in [14] a simultaneously computation for the observer and controller gains. The authors reduced the conservatism introduced by the two computing steps. They could have simultaneously different performances requirements on the observer and the controller gains (settling time, disturbance attenuation, for example). Their method was first based on Lyapunov theory with the use of the DMVT (differential mean value theorem). Then it went toward the solution of the bilinear matrix inequality (BMI) to prove the stability of the system. After that, they provided new LM conditions which help in the H_∞ stabilization of a class of nonlinear discrete time systems. They utilized some suitable algebraic transformations (Schur lemma and Young inequality) on the BMI which contains the observer and the controller gains. In [2], the authors intend to deal with the observer-based H_∞ control design problem for a special class of nonlinear networked control systems with both random sensor-to-controller and controller-to-actuator packet losses. They considered the packet loss for both ways of communications be different. They modeled the packet loss as two cases either occurred or not occurred using Bernoulli distribution. The design of the controller and the observer gains were found by solving an LMI and an MI. To solve LMI and MI, they suggested that definite symmetric matrices of LLM(Lyapunov Like Method) P and Q are the same which is not correct in every case. In this paper, the design of the controller of NNCS will be done on the existing of the packet loss in both directions. The type of the NNCS controller will be an H_∞ observer-based. The analysis will follow first the work in [2], but the final result is to solve an LMI with different Lyapunov Like matrices P and Q. Finally, a simulation example will be presented to demonstrate the effectiveness of the proposed LMI approach. Comparison with existing results will be presented.

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Nomenclature:

Pr$\{\cdot\}$ means the occurrence probability of the event "$\cdot$". $\mathbb{E}\{\boldsymbol{x}\}$ means the expectation of the stochastic variable $\boldsymbol{x}$. $\boldsymbol{l_2[0,\infty)}$means the space of square integral vectors. $\mathbb{I}^+$means the set of the positive integers. $\mathbb{R}$means the real numbers and $\mathbb{R}^{\boldsymbol{n}}$ means the $\boldsymbol{n}$ -dimensional of Euclidean space. $\mathbb{R}^{\boldsymbol{n\times m}}$ means the set of all $\boldsymbol{n\times m}$ real matrices. $\boldsymbol{\lambda_{max}(A)}$ and $\boldsymbol{\lambda_{min}(A)}$ mean the maximum eigenvalue of matrix $\boldsymbol{A}$ and the minimum eigenvalue of matrix $\boldsymbol{A}$ respectively. $\|\cdot\|$means Euclidean vector norm or the induced matrix $\boldsymbol{2-norm}$. $\boldsymbol{I}$ means the identity matrix with appropriate dimension. $\boldsymbol{X > Y}$or$(\boldsymbol{X \geq Y})$ means $\boldsymbol{X - Y}$ are definite matrix (semi-definite matrix) where $\boldsymbol{X}$ and $\boldsymbol{Y}$ are symmetric matrices. $\boldsymbol{diag(a_1, a_2, \ldots\ldots a_n)}$means block-diagonal matrix."$*$" is used as an ellipsis for terms induced by the symmetry in symmetric block matrices.

II. PROBLEM FORMULATION

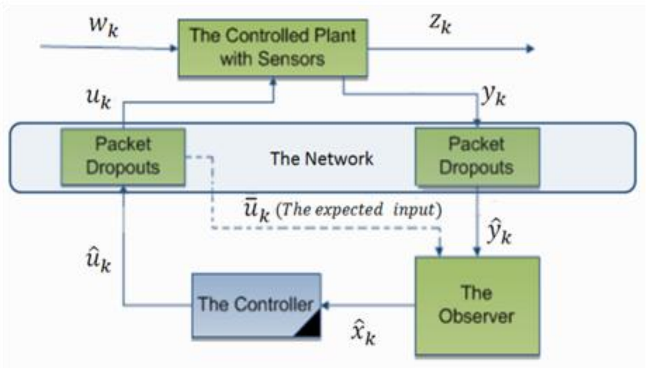


Figure 1. The layout of the NCS system

The configuration of the NCSs considering packet losses is shown in figure 1.The controlled plant is a nonlinear system. The random packet losses occur, simultaneously, in the communication channels both from the sensor to the controller and from the controller to the actuator. It is supposed that the data is transmitted in single-packetmanner with the same transmission length, and employ the point-to-point network allowable data dropout rate. Also, it is suggested that the network is point-to-point throughout to evaluate the QoS of the investigated networked nonlinear system. Consider the following networked nonlinear control systems after sampling:

$$\begin{aligned} x_{k+1} &= Ax_k + f(k, x_k) + Bu_k + Dw_k \\ z_{k+1} &= C_1 x_k + D_1 w_k \\ y_k &= \alpha_k C_2 x_k + D_2 w_k \end{aligned} \qquad (1)$$

where $x_k \in \mathbb{R}^n$is the state,$u_k \in \mathbb{R}^m$is the control input,$z_k \in \mathbb{R}^r$ is the controlled output, $\hat{y}_k \in \mathbb{R}^p$ is the measured output, $w_k \in \mathbb{R}^q$ is the disturbance input belong to $l_2[0,\infty)$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $D \in \mathbb{R}^{n\times q}$,$C_1 \in \mathbb{R}^{r\times n}$,$C_2 \in \mathbb{R}^{p\times n}$,$D_1 \in \mathbb{R}^{r\times q}$,$D_2 \in \mathbb{R}^{p\times q}$are known matrices.$f(k, x_k)$is nonlinear vector function satisfies the global Lipschitz condition:

$$\|f(k,x)\| \leq \|Gx\| \qquad (2)$$

$$\|f(k,x) - f(k,y)\| \leq \|G(x-y)\| \qquad (3)$$

where G is a known real constant matrix. The stochastic variable $\alpha_k \in \mathbb{R}$ represent the Bernoulli distribution of packet loss from the sensor to the controller and it has been considered as linear stochastic variable sequence with the following properties [2]

$$\Pr\{\alpha_k = 1\} = \mathbb{E}\{\alpha_k\} = \bar{\alpha} \qquad (4)$$

$$\Pr\{\alpha_k = 0\} = 1 - \mathbb{E}\{\alpha_k\} = 1 - \bar{\alpha} \qquad (5)$$

$$\begin{aligned} \mathrm{Var}\{\alpha_k\} &= \mathbb{E}\{(\alpha_k - \bar{\alpha})^2\} = (1-\bar{\alpha})\bar{\alpha} \\ &= \alpha_1^2 \end{aligned} \qquad (6)$$

The nonlinear dynamic observer considered here is given as

$$\begin{aligned} \hat{x}_{k+1} &= A\hat{x}_k + f(k, \hat{x}_k) + B\bar{u}_k + L(\hat{y}_k - \bar{\alpha}C_2\hat{x}_k) \\ \bar{u}_k &= \bar{\beta}\hat{u}_k \end{aligned} \qquad (7)$$

and the controller

$$\begin{aligned} \hat{u}_k &= -K\hat{x}_k \\ u_k &= \beta_k \hat{u}_k \end{aligned} \qquad (8)$$

where $\hat{x}_k \in \mathbb{R}^n$ is the state estimate of the networked nonlinear system (1),$\bar{u}_k \in \mathbb{R}^m$is the control input to the observer, $\hat{u}_k \in \mathbb{R}^m$ is the control input without random packet loss, $u_k \in \mathbb{R}^m$is the control input of the controlled system, $L \in \mathbb{R}^{n\times p}$is the observer gain and $K \in \mathbb{R}^{m\times n}$is the controller gain. The stochastic variable$\beta_k \in \mathbb{R}$ is a Bernoulli distributed white sequence represent thepacket loss from the controller to the actuator with

$$\Pr\{\beta_k = 1\} = \mathbb{E}\{\beta_k\} = \bar{\beta} \qquad (9)$$

$$\Pr\{\beta_k = 0\} = 1 - \mathbb{E}\{\beta_k\} = 1 - \bar{\beta} \qquad (10)$$

$$\mathrm{Var}\{\beta_k\} = \mathbb{E}\left\{\left(\beta_k - \bar{\beta}\right)^2\right\} = \left(1 - \bar{\beta}\right)\bar{\beta} = \beta_1^2 \qquad (11)$$

$\beta_k$is considered as a linear stochastic variable.
Let $e_k$represent the error in the states described by

$$e_k = x_k - \hat{x}_k \qquad (12)$$

Then, the nonlinear closed-loop networked states are

$$\begin{aligned} x_{k+1} = \left(A - \bar{\beta}BK\right)x_k + \left(\beta_k - \bar{\beta}\right)BKe_k - \left(\beta_k - \bar{\beta}\right)BKx_k + \bar{\beta}BKe_k \\ + f(k, x_k) + Dw_k \end{aligned} \qquad (13)$$

$$\hat{x}_{k+1} = \left(A - \bar{\beta}BK\right)\hat{x}_k + f(k, \hat{x}_k) + L(\hat{y}_k - \bar{\alpha}C_2\hat{x}_k) \qquad (14)$$

Let $e_{k+1} = x_{k+1} - \hat{x}_{k+1}$, then

$$e_{k+1} = -(\beta_k - \bar{\beta})BKx_k - (\alpha_k - \bar{\alpha})LC_2x_k + (A - \bar{\alpha}LC_2)e_k + (\beta_k - \bar{\beta})BKe_k + f(k, x_k) - f(k, \hat{x}_k) + (D + LD_2)w_k \quad (15)$$

The nonlinear closed-loop networked system can be written as

$$\eta_{k+1} = \bar{A}\eta_k + (\beta_k - \bar{\beta})\hat{A}_1\eta_k + (\alpha_k - \bar{\alpha})\hat{A}_2\eta_k + \bar{F}_k + \bar{B}w_k \quad (16)$$

where $\eta_k^t = [x_k^t \quad e_k^t]^t$, $\bar{A} = \begin{bmatrix} A + \bar{\beta}BK & \bar{\beta}BK \\ 0 & A - \bar{\alpha}LC_k \end{bmatrix}$, $\bar{F}_k = \begin{bmatrix} f(k, x_k) \\ F_k \end{bmatrix}$, $\hat{A}_2 = \begin{bmatrix} 0 & 0 \\ -LC_2 & 0 \end{bmatrix}$,

$$\bar{B} = \begin{bmatrix} D \\ D + LD_2 \end{bmatrix}, F_k = f(k, x_k) - f(k, \hat{x}_k).$$

In this paper, the objective is designing the observer (7) and the controller (8) for the nonlinear networked system (1), such that, in the presence of the random packet losses and disturbance, the closed-loop nonlinear networked system (16) is exponentially mean square stable and its H_∞-performance constraint is achieved.

III. MAIN RESULTS

The aim of this work is to design a controller such that the closed loop nonlinear networked system (16) satisfies the following two requirements

1-The closed -loop nonlinear networked system (16) is exponentially mean square stable

2-Under the zero-initial condition and for all nonzero white noise w_k ; the controlled output z_k should satisfy the condition

$$\sum_{k=0}^{\infty} \mathbb{E}\|z_k\|^2 < \gamma^2 \sum_{k=0}^{\infty} \mathbb{E}\|w_k\|^2 \quad (17)$$

where $\gamma > 0$ is a prescribed scalar .

In this section, the main results will be presented. The following definition and lemma will be used.

Definition 1: The closed loop nonlinear networked system (16) is said to be exponentially mean-square stable, when $w_k = 0$, if there exist constant $\phi > 0$ and $\tau\epsilon(0,1)$ such that

$$\mathbb{E}\{\|\eta_k\|^2\} \leq \emptyset\tau^k\mathbb{E}\{\|\eta_o\|^2\}, \quad \forall\eta_o \in \mathbb{R}^n, k \in I^+ \quad (18)$$

Lemma 1:([17,18](S-procedure). Let $T_i \in \mathbb{R}^{n\times n}(i = 0,1,2,\dots\dots,p)$ be symmetric matrices. The condition on $T_i(i = 0,1,2,\dots\dots,p)$, $\varsigma^T T_o\varsigma > 0$, $\forall\varsigma \neq 0$ s.t. $\varsigma^T T_i\varsigma \geq 0\ (i = 0,1,2,\dots\dots,p)$ hold if there exist $\tau_i \geq 0\ (i = 0,1,2,\dots\dots,p)$ such that $T_o - \sum_{i=0}^{p}\tau_i T_i > 0$.

The following theorem provides a sufficient condition such that the closed-loop nonlinear networked system (16) is exponentially mean square stable and achieve the H_∞- performance constraint(17).

Theorem 1: Given communication channel parameters $0 \leq \bar{\alpha} \leq 1, 0 \leq \bar{\beta} \leq 1$ and a scalar $\gamma > 0$. The closed loop nonlinear networked system (16) is exponentially mean square stable and the H_∞- performance constraint (17) is achieved for all nonzero w_k , if there exist positive definite matrices $P > 0, Q > 0$, real matrices K , L and real scalars $\tau_1 > 0, \tau_2 > 0$ satisfying the following matrix inequality

$$\begin{bmatrix} \Pi_{11} & * \\ \Pi_{21} & \Pi_{22} \end{bmatrix} < 0 \quad (19)$$

where $\Pi_{11} = \begin{bmatrix} \tilde{\Phi}_{11} & * & * & * & * \\ 0 & \tilde{\Phi}_{22} & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * \\ 0 & 0 & 0 & -\tau_1 I & * \\ 0 & 0 & 0 & 0 & -\tau_2 I \end{bmatrix}$, $\Pi_{21} = \begin{bmatrix} \Phi_{61} & \bar{\beta}PBK & PD & P & 0 \\ 0 & \Phi_{72} & \Phi_{73} & 0 & Q \\ C_1 & 0 & D_1 & 0 & 0 \\ -\beta_1 PBK & \beta_1 PBK & 0 & 0 & 0 \\ -\beta_1 QBK & \beta_1 QBK & 0 & 0 & 0 \\ \alpha_1 QLC_2 & 0 & 0 & 0 & 0 \end{bmatrix}$,

$$\Pi_{22} = \begin{bmatrix} -P & * & * & * & * & * \\ 0 & -Q & * & * & * & * \\ 0 & 0 & -I & * & * & * \\ 0 & 0 & 0 & -P & * & * \\ 0 & 0 & 0 & 0 & -Q & * \\ 0 & 0 & 0 & 0 & 0 & -Q \end{bmatrix},$$

$$\tilde{\Phi}_{11} = -P + \tau_1 G^T G,$$

$$\tilde{\Phi}_{22} = -Q + \tau_2 G^T G,$$

$$\tilde{\Phi}_{22} = -Q + \tau_2 G^T G,$$

$$\Phi_{61} = P(A - \bar{\beta}BK),$$

$$\Phi_{72} = QA - \bar{\alpha}QLC_2,$$

$$\Phi_{73} = QD - QLD_2,$$

$$\alpha_1 = [(1-\bar{\alpha})\bar{\alpha}]^{\frac{1}{2}} \text{and } \beta_1 = \left[(1-\bar{\beta})\bar{\beta}\right]^{\frac{1}{2}}.$$

Proof: Using a Lyapunov function

$$V_k = x_k^T P x_k + e_k^T Q e_k \qquad (21)$$

where P, Q are positive definite matrices ,then

$$\Delta V_k = \mathbb{E}\{V_{k+1} | x_k, x_{k-1}, x_{k-2}, \dots \dots \dots \dots, x_0, e_k, e_{k-1}, e_{k-2}, \dots \dots \dots .., e_0\} - V_k \qquad (22)$$

which can be written as

$$\Delta V_k = \mathbb{E}\{x_{k+1}^T P x_{k+1} + e_{k+1}^T Q e_{k+1}\} - x_k^T P x_k - e_k^T Q e_k \qquad (23)$$

Using (13) and (15), the properties of the expecting value of the Bernoulli distribution $\mathbb{E}\left\{(\beta_k - \bar{\beta})^2\right\} = (1-\bar{\beta})\bar{\beta} = \beta_1^2$, $\mathbb{E}\{(\alpha_k - \bar{\alpha})^2\} = \alpha_1^2$, $\mathbb{E}\{(\beta_k - \bar{\beta})\} = \mathbb{E}(\beta_k) - \bar{\beta} = \bar{\beta} - \bar{\beta} = 0$, $\mathbb{E}\{(\alpha_k - \bar{\alpha})\} = 0$, $w_k = 0$, with simple matrix manipulation and arrangements , ΔV_k can be written as

$$\Delta V_k \triangleq \xi_k^T \Lambda \xi_k \qquad (24)$$

Where $\xi_k^T = [x_k^t \quad e_k^t \quad f(k, w_k) \quad F_k^t]^t$,

$$\Lambda = \begin{bmatrix} -P + \Phi_{11} & * & * & * \\ \Phi_{21} & -Q + \Phi_{22} & * & * \\ P(A - \bar{\beta}BK) & \bar{\beta}PBK & P & * \\ 0 & Q(A - \bar{\alpha}LC_2) & 0 & Q \end{bmatrix},$$

$\Phi_{11} = (A - \bar{\beta}BK)^T P (A - \bar{\beta}BK) + \beta_1^2 K^T B^T PBK + \beta_1^2 K^T B^T QBK + \alpha_1^2 C_2^T L^T QLC_2$,

$\Phi_{22} = \bar{\beta}^2 K^T B^T PBK + \beta_1^2 K^T B^T PBK + \beta_1^2 K^T B^T QBK + (A - \bar{\alpha}LC_2)^T Q (A - \bar{\alpha}LC_2)$ and

$\Phi_{21} = \bar{\beta} K^T B^T P (A - \bar{\beta}BK) - \beta_1^2 K^T B^T PBK - \beta_1^2 K^T B^T QBK$

The constraints in (2) and (3) on the nonlinear functions $f(k; x_k)$ and F_k can be rewritten as

$$\xi_k^T \begin{bmatrix} -G^T G & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xi_k \triangleq \xi_k^T \Lambda_1 \xi_k \leq 0 \qquad (25)$$

and

$$\xi_k^T \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -G^T G & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xi_k \triangleq \xi_k^T \Lambda_1 \xi_k \leq 0 \qquad (26)$$

By using Lemma 1, the following inequality holds if there exist P, Q and scalars $\tau_1 > 0$; $\tau_2 > 0$ such that $\Lambda - \tau_1 \Lambda_1 - \tau_2 \Lambda_2 < 0$. Then , we get the following result

$$\begin{bmatrix} -P + \tau_1 G^T G & * & * & * \\ 0 & -Q + \tau_1 G^T G & * & * \\ 0 & 0 & -\tau_1 I & * \\ 0 & 0 & 0 & -\tau_2 I \end{bmatrix} + \begin{bmatrix} \Phi_{11} & * & * & * \\ \Phi_{21} & \Phi_{22} & * & * \\ P(A - \bar{\beta}BK) & \bar{\beta}PBK & P & * \\ 0 & Q(A - \bar{\alpha}LC_2) & 0 & Q \end{bmatrix} < 0 \qquad (27)$$

Using Schur complement on (27) we get

$$\begin{bmatrix} \tilde{\Pi}_{11} & * \\ \tilde{\Pi}_{21} & \tilde{\Pi}_{22} \end{bmatrix} < 0 \qquad (28)$$

where

$\tilde{\Pi}_{11} = \begin{bmatrix} -P+\tau_1 G^T G & * & * & * \\ 0 & -Q+\tau_1 G^T G & * & * \\ 0 & 0 & -\tau_1 I & * \\ 0 & 0 & 0 & -\tau_2 I \end{bmatrix}$, $\tilde{\Pi}_{21} = \begin{bmatrix} P(A-\bar{\beta}BK) & \bar{\beta}BK & P & 0 \\ \beta_1 PBK & -\beta_1 PBK & 0 & 0 \\ \beta_1 QBK & -\beta_1 QBK & 0 & 0 \\ \alpha_1 QLC_2 & 0 & 0 & 0 \\ 0 & Q(A-\bar{\alpha}LC_2) & 0 & Q \end{bmatrix}$ and

$\tilde{\Pi}_{22} = \begin{bmatrix} -P & * & * & * & * \\ 0 & -Q & * & * & * \\ 0 & 0 & -P & * & * \\ 0 & 0 & 0 & -Q & * \\ 0 & 0 & 0 & 0 & -Q \end{bmatrix}$.

Following discussion of [2], we have

$\Delta V_k = \xi_k^T \Lambda \xi_k < 0$ since $\Lambda < 0$

$\Delta V_k = \xi_k^T \Lambda \xi_k \le -\lambda_{\min}(-\Lambda)\xi_k^T \xi_k$

$\Delta V_k \le -\lambda_{\min}(-\Lambda)(\eta_k^T \eta_k + f(k,x_k)^T f(k,x_k) + F_k^T F_k)$

$\Delta V_k \le -\lambda_{\min}(-\Lambda)(\eta_k^T \eta_k + \|f(k,x_k)\|^2 + \|F_k\|^2) < -\alpha \eta_k^T \eta_k$

where $0 < \alpha < \min\{\lambda_{min}(-\Lambda), \sigma\}$, $0 < \alpha < \min\{\lambda_{min}(-\Lambda), max\{\lambda_{max}(P), \lambda_{max}(Q)\}\}$. This will lead to the following

$$\Delta V_k < -\alpha \eta_k^T \eta_k < -\frac{\alpha}{\sigma} V_k := -\psi V_k \qquad (30)$$

Therefore by definition (18), it is verified from Theorem 1 that the closed-loop nonlinear networked system (16) is exponentially mean-square stable.For non-zero$w_k$and the outputz_k, the following equation holds

$$\begin{aligned} \mathbb{E}\{V_{k+1}\} - \mathbb{E}\{V_k\} + \mathbb{E}\{Z_k^T Z_k\} - \gamma^2 \mathbb{E}\{w_k^T w_k\} \\ = \mathbb{E}\{x_{k+1}^T P x_{k+1} + e_{k+1}^T Q e_{k+1}\} - x_k^T P x_k - e_k^T Q e_k \\ + [C_1 x_k + D_1 w_k]^T [C_1 x_k + D_1 w_k] - \gamma^2 \mathbb{E}\{w_k^T w_k\} \end{aligned} \qquad (31)$$

Simplifying the previous expression as in the proof of Theorm1,we get

$$\mathbb{E}\{V_{k+1}\} - \mathbb{E}\{V_k\} + \mathbb{E}\{Z_k^T Z_k\} - \gamma^2 \mathbb{E}\{w_k^T w_k\} \triangleq \zeta_k^T \Omega \zeta_k \qquad (32)$$

where $\Omega = \begin{bmatrix} \Psi_{11} & * & * & * & * \\ \Psi_{21} & \Psi_{22} & * & * & * \\ \Psi_{31} & \Psi_{32} & \Psi_{33} & * & * \\ \Psi_{41} & \bar{\beta}PBK & PD & P & * \\ 0 & \Psi_{52} & Q(D-LD_2) & 0 & Q \end{bmatrix}$,

$\zeta_k^T = [x_k^T \quad e_k^T \quad w_k^T \quad f^T(k,x_k) \quad F_k^T]^T$,

$\Psi_{11} = (A-\bar{\beta}BK)^T P (A-\bar{\beta}BK) + \beta_1^2 K^T B^T PBK + \beta_1^2 K^T B^T QBK + \alpha_1^2 C_2^T L^T QLC_2 + C_1^T C_k^T - P$,

$\Psi_{22} = \bar{\beta}^2 K^T B^T PBK + \beta_1^2 K^T B^T PBK + \beta_1^2 K^T B^T QBK + (A - \bar{\alpha} LC_2)^T Q(A - \bar{\alpha} LC_2) - Q,$

$\Psi_{21} = \bar{\beta} K^T B^T P(A - \bar{\beta} BK) - \beta_1^2 K^T B^T PBK - \beta_1^2 K^T B^T QBK$,

$\Psi_{33} = D^T PD + (D - LD_2)^T Q(D - LD_2) + D^T D_1 - \gamma^2 I,$

$\Psi_{31} = \bar{\beta} D^T PBK + (D - LD_2)^T Q(A - \bar{\alpha} LC_2),$

$\Psi_{31} = D^T P(A - \bar{\beta} BK) + D_1^T C_1,$

$\Psi_{41} = PA - \beta PBK$ and

$\Psi_{52} = Q(A - \bar{\alpha} LC_2).$

The two constraint(2) and (3) can be rewritten as

$$\zeta_k^T \begin{bmatrix} -G^T G & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \zeta_k \triangleq \zeta_k^T \Omega_1 \zeta_k \ \leq 0 \text{and}$$

$$\zeta_k^T \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -G^T G & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix} \zeta_k \triangleq \zeta_k^T \Omega_2 \zeta_k \ \leq 0.$$

By the S-procedure, the inequality

$$\Omega - \tau_1 \Omega_1 - \tau_2 \Omega_2 < 0 \qquad (33)$$

holds if there exist positive-definite matrices P, Q and nonnegative scalars $\tau_1 > 0; \tau_2 > 0$.After a rearrangement, it gives the same MI shown in (19).It can be concluded from (32) that

$$\mathbb{E}\{V_{k+1}\} - \mathbb{E}\{V_k\} + \mathbb{E}\{Z_k^T Z_k\} - \gamma^2 \mathbb{E}\{w_k^T w_k\} < 0 \qquad (34)$$

Now , summing (34) from $0 \ to \ \infty$ with respect to k yeild

$$\sum_{k=0}^{\infty} \mathbb{E}\{Z_k^T Z_k\} < \gamma^2 \sum_{k=0}^{\infty} \mathbb{E}\{w_k^T w_k\} - \mathbb{E}\{V_0\} - \mathbb{E}\{V_\infty\} \qquad (35)$$

Since the closed loop nonlinear network system (16) is exponentially mean square stable and $\zeta_0 = 0$,it is straight forward to conclude that

$$\sum_{k=0}^{\infty} \mathbb{E}\{Z_k^T Z_k\} < \gamma^2 \sum_{k=0}^{\infty} \mathbb{E}\{w_k^T w_k\} \qquad (36)$$

which means the specified H_∞- performance constraints (17) is achieved. This complete the proof.

IV. CONTROLLER DESIGN

The MI given in (19) is not linear in P, K, Q and L.Pre and post multiply (19) by the congruence transformation matrix $diag\{I \quad I \quad I \quad I \quad I \quad P^{-1} \quad Q^{-1} \quad I \quad P^{-1} \quad Q^{-1} \quad Q^{-1}\}$ to get

$$\begin{bmatrix} \Pi_{11} & * \\ \widetilde{\Pi}_{21} & \widetilde{\Pi}_{22} \end{bmatrix} < 0 \qquad (37)$$

where Π_{11}as in (19) ,

$$\widetilde{\Pi}_{21} = \begin{bmatrix} U_{61} & \bar{\beta} BK & D & I & 0 \\ 0 & U_{72} & U_{73} & 0 & I \\ C_1 & 0 & D_1 & 0 & 0 \\ -\beta_1 BK & \beta_1 BK & 0 & 0 & 0 \\ -\beta_1 BK & \beta_1 BK & 0 & 0 & 0 \\ \alpha_1 LC_2 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$\widetilde{\Pi}_{22} = -diag\{P^{-1} \quad Q^{-1} \quad I \quad P^{-1} \quad Q^{-1} \quad Q^{-1}\},$

$U_{61} = (A - \bar{\beta} BK),$

$U_{72} = A - \bar{\alpha} LC_2$and

$U_{73} = D - LD_2.$

Two approaches will be proposed to solve for K and L. In eachapproach an optimization problem will be solved.

4.1 First Approach

This approach is based on using the identity $-X^{-1} < -2I + X$, then $\widetilde{\Pi}_{22}$ in (37) becomes

$$\widetilde{\Pi}_{22} = diag\{U_{81} \quad U_{91} \quad -I \quad U_{81} \quad U_{91} \quad U_{91}\} \qquad (38)$$

where $U_{81} = -2I + P$,$U_{91} = -2I + Q$ and the following optimization problem will be solved

$$\min_{P>0, Q>0, K, L, \tau_1>0, \tau_2>0} \gamma \qquad (39)$$

subject to the inequality given in (37) with $\widetilde{\Pi}_{22}$, as given in (38).

4.2 Second Approach

This approach is based on the S- procedure$\widetilde{\Pi}_{22} - \Omega_3 < 0$,where

$\Omega_3 = diag\{N_1 \quad N_2 \quad 0 \quad N_1 \quad N_2 \quad N_2\} > 0$,$N_1, N_2$ are matrices with the same dimensions of P and Q respectively. The matrices N_1, N_2 are found by the following two equations $N_1 = (\widetilde{N}_1 - P^{-1})$ and $N_2 = (\widetilde{N}_2 - Q^{-1})$ where $\widetilde{N}_1$ and $\widetilde{N}_2$ are positive- definite matrices. Therefore

$$\widetilde{\Pi}_{22} - \Omega_3 = diag\{-\widetilde{N}_1 \quad -\widetilde{N}_2 \quad -I \quad -\widetilde{N}_1 \quad -\widetilde{N}_2 \quad -\widetilde{N}_2\} \qquad (40)$$

and the following optimization problem will be solved

$$\min_{P>0,Q>0,\tilde{N}_1>0,\tilde{N}_2>0,K,L,\tau_1>0,\tau_2>0} \gamma \quad (41)$$

subject to inequality given in (37) with $\widetilde{\Pi}_{22}$, as given in (40).

V. NUMERICAL EXAMPLE

This section demonstrate the results presented in this paper. The optimization problems stated in (39) and (41) will be solved using Matlab ToolBox.Considering a system described by (1), the following parameter from [4] are used.

$$A = \begin{bmatrix} 0.8226 & -0.633 & 0 \\ 0.5 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \; D = \begin{bmatrix} 0.5 \\ 0 \\ 0.2 \end{bmatrix}$$

$$C_1 = [0.1 \quad 0 \quad 0], \qquad C_2 = [23.738 \quad 20.287 \quad 0], D_1 = 0.1 \quad D_2 = 0.2$$

$$f(k, x_k) = \begin{bmatrix} 0.01\sin & x_k^1 \\ 0.01\sin & x_k^2 \\ 0.01\sin & x_k^3 \end{bmatrix} x_k = \begin{bmatrix} x_k^1 \\ x_k^2 \\ x_k^3 \end{bmatrix} \text{and} G = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}$$

The initial conditions of the networked nonlinear system (1) have been assumed as $x_0^T = [0.2 \quad 0.3 \quad 0.1]^T$. $\hat{x}_0^T = [0 \quad 0 \quad 0]^T$, and the disturbance input was taken as $\omega_k = 1/k^2$.The controller (8) for system (1) has been designed such that the H_∞- performance index was minimized.

In this example, three values are taken for each of $\bar{\alpha}$ and $\bar{\beta}$ to study the effect of different probability of data dropout through the network.

The first trail was made at $\bar{\alpha} = 0.95$ and $\bar{\beta} = 0.9$ and the following results have been obtained:

1- Using first approach

$$K = [0.3739 \quad -0.2877 \quad 0], L = \begin{bmatrix} 0.0070 \\ 0.0122 \\ 0.0208 \end{bmatrix}, \gamma_{\min.} = 4.9034.$$

2-Using second approach

$$K = [0.3739 \quad -0.2877 \quad 0], L = \begin{bmatrix} 0.0070 \\ 0.0122 \\ 0.0208 \end{bmatrix}, \gamma_{\min.} = 3.3196.$$

The second trail was made at $\bar{\alpha} = 0.7$ and $\bar{\beta} = 0.65$ and the following results have been obtained:

1-Using first approach

$$K = [0.3047 \quad -0.2344 \quad 0], L = \begin{bmatrix} 0.0070 \\ 0.0122 \\ 0.0209 \end{bmatrix} \gamma_{\min.} = 4.9194$$

2- Using second approach

$$K = [0.3047 \quad -0.2344 \quad 0], L = \begin{bmatrix} 0.0070 \\ 0.0122 \\ 0.0209 \end{bmatrix} \gamma_{\min.} = 3.3418$$

The third trail was made at $\bar{\alpha} = 0.65$ and $\bar{\beta} = 0.60$ and the following results have been obtained:

1-Using first approach

$$K = [0.2938 \quad -0.2261 \quad 0], L = \begin{bmatrix} 0.007 \\ 0.0122 \\ 0.0209 \end{bmatrix} \gamma_{\min.} = 4.9222$$

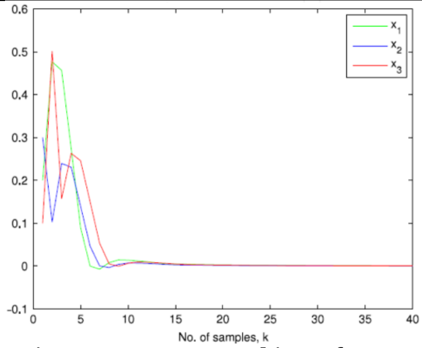
2- Using second approach

$$K = [0.2938 \quad -0.2261 \quad 0], L = \begin{bmatrix} 0.007 \\ 0.0122 \\ 0.0209 \end{bmatrix} \gamma_{\min.} = 3.3457$$

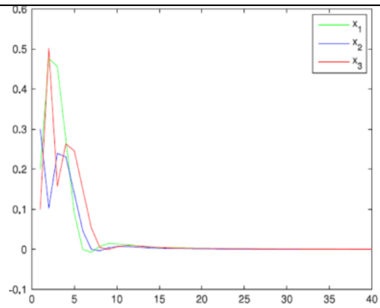
A comparison between the different types of response are shown in Figures 2-10. It is clear that the system is exponentially mean square stable. A comparison has been made between the results of these two approaches and the results reported in [2] is shown in the following Table1.

Table 1: Comparison between the reference [2] and the two approaches (39) and (40)

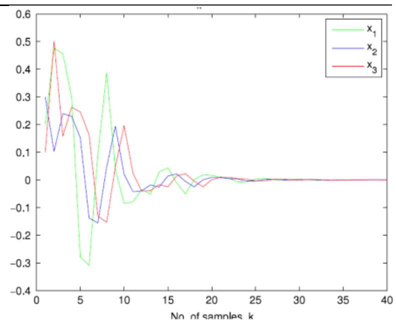
		source[4]		First approach(39)		second approach (40)	
$\bar{\alpha}$	$\bar{\beta}$	$\gamma_{\min}$	Figure#	$\gamma_{\min}$	Figure#	$\gamma_{\min}$	Figure#
0.95	0.9	0.4574	2	4.9034	5	3.3196	8
0.7	0.65	2.2357	3	4.9194	6	3.3418	9
0.65	0.6	8.9523	4	4.9222	7	3.3457	10



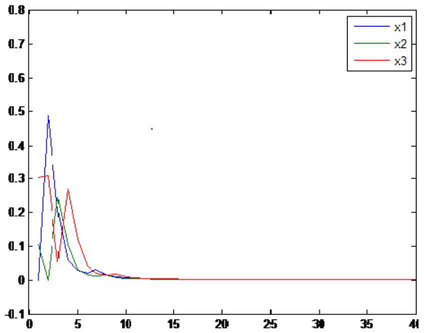
Figure#2: H_∞ Control in Reference [2] at $\bar{\alpha} = 0.95$ & $\bar{\beta} = 0.9$



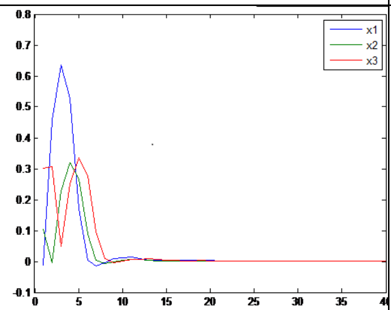
Figure#3: H_∞ Control in Reference [2] at $\bar{\alpha} = 0.7$ & $\bar{\beta} = 0.65$



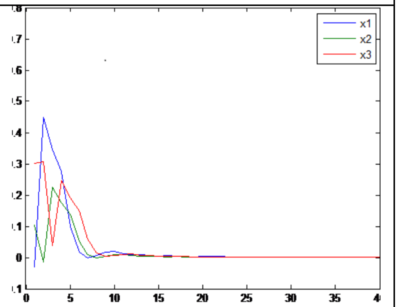
Figure#4: H_∞ Control in Reference [2] at $\bar{\alpha} = 0.65$ & $\bar{\beta} = 0.6$



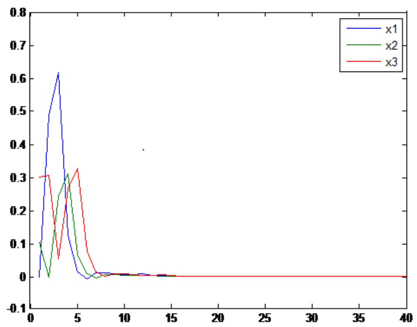
Figure#5: H_∞ Control by 1st approach (39) at $\bar{\alpha} = 0.95$ & $\bar{\beta} = 0.9$



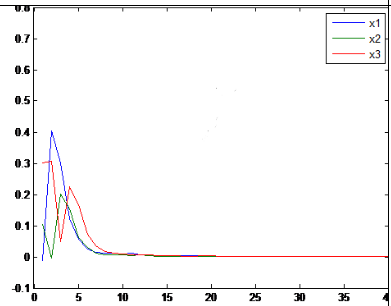
Figure#6: H_∞ Control by 1st approach (39) at $\bar{\alpha} = 0.7$ & $\bar{\beta} = 0.65$



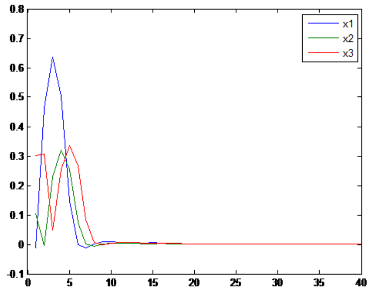
Figure#7: H_∞ Control by 1st approach (39) at $\bar{\alpha} = 0.65$ & $\bar{\beta} = 0.6$



Figure#8: H_∞ Control by 2nd approach (40) at $\bar{\alpha} = 0.95$ & $\bar{\beta} = 0.9$



Figure#9: H_∞ Control by 2nd approach (40) at $\bar{\alpha} = 0.7$ & $\bar{\beta} = 0.65$



Figure#10: H_∞ Control by 2nd approach (40) at $\bar{\alpha} = 0.65$ & $\bar{\beta} = 0.6$

It is obvious from the table that the LMI approach in source[2] can be proper for low packet loss probability, but it will become higher in the high probability of packet loss and the same thing in its stability. Even though the performance index in the new two approaches are higher than the LMI which is in source [2] for the low probability of packet loss, they can keep acceptable average values in the high probability of the packet loss. It is noticed that both of them are very near to each other in the stability behaviour but the performance index of the second approach design is minimized more than that of the first approach. Both of them keep the balanced values of the performance for the probabilities of packet loss.

VI. CONCLUSION

At the end of this paper it can be concluded that an observer-based H_∞controller has been designed for a class of the NNCS. The packet loss change has been described as probability with Bernoulli distribution and the probability of the packet loss from the sensor to the controller was different from that of the controller to the actuator. Certain conditions have succeeded to make the system work stably in exponentially mean-square sense. The numerical example explains the effects of these new modifications on the system stability and what was their level comparing to the source [2].

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