Fuzzy Inventory Model for Power Demand Pattern with Shortages, Inflation under Permissible Delay in Payment

Wasim Akram Mandal, Sahidul Islam

***Abstract:** In this paper fuzzy inventory model for non deteriorating item with power demand pattern, shortage under partially backlogged, inflation and consideration of permissible delay in payment is formulated and solved. Fuzziness is applying by allowing the cost components (holding cost, shortage cost, etc) and inflation. In fuzzy environment it considered all required parameter to be pentagonal fuzzy numbers. The purpose of the model is to minimize total cost function.*

***Keywords: -** Inventory, Power demand, Fuzzy number, Shortages, Inflation, Pentagonal fuzzy number*

I. INTRODUCTION

An inventory deal with decision that minimum the total average cost or maximize The total average profit. For this purpose the task is to construct a mathematical model of the real life Inventory system, such a mathematical model is based on various assumption and approximation In a inventory model shortages is very important condition. There are several type of customer. At shortage period some customers are waiting for actual product and others do not it. For this it consider partially- backlogging. In ordinary inventory model it consider all parameter like shortage cost, holding cost, unit cost as fixed. But in real life situation it will have some little fluctuations. So consideration of fuzzy variables is more realistic. The study of inventory model where demand rates varies with time is the last decades. Datta and pal investigated an inventory system with power demand pattern for item which variable rate deterioration. Park and Wang studied shortages and partial backlogging of items. Friedman(1978) presented continuous time inventory model with time varying demand. Ritchie(1984) studied in inventory model with linear increasing demand. Goswami, Chaudhuri(1991) discussed an inventory model with shortage.Gen et. Al. (1997) considered classical inventory model with Triangular fuzzy number.Yao and Lee(1998) considered an economic production quantity model in the fuzzy sense. Sujit Kumar De, P.K.Kundu and A.Goswami(2003) presented an economic production quantity inventory model involving fuzzy demand rate. J.K.Syde and L.A.Aziz(2007) applied sign distance method to fuzzy inventory model without shortage . D. Datta and Pravin Kumar published several paper of fuzzy inventory with or without shortage.

In this paper we first consider crisp inventory model with power demand where shortage are allowed and partially backlogged. Thereafter we developed fuzzy inventory model with fuzzy power demand rate under partially backlogged. All inventory cost parameters are fuzzy fied as pentagonal fuzzy number.

II. PRELIMINARIES

For graded representation method to defuzzyfy, we need the following definitions,

Definition2.1: A fuzzy set $\tilde{A}$ on the given universal set X is a set of order pairs,

$\tilde{A}=\{(x,\mu_A(x)): x\epsilon X\}$ where $\mu_A(x)\rightarrow[0,1]$ is called a membership function.

Definition2.2:The α-cut of $\tilde{A}$, is defined by $A_\alpha=\{x: \mu_A(x)=\alpha, \alpha\geq 0\}$

Definition2.3:$\tilde{A}$ is normal if there exists xϵX such that $\mu_A(x)=1$

Definition2.4:A pentagonal fuzzy number $\tilde{A}$=(a,b,c,d,e) is represented with membership function $\tilde{A}$

$\tilde{A}$ is defined as,

$$\mu_A(X)=\begin{cases} L_1(x)=\frac{x-a}{b-a}, a\leq x\leq b \\ L_2(x)=\frac{x-b}{c-b}, b\leq x\leq c \\ 1, \quad x=c \\ R_1(x)=\frac{d-x}{d-c}, c\leq x\leq d \\ R_2(x)=\frac{e-x}{e-d}, d\leq x\leq e \\ 0 \quad, otherwise \end{cases}$$

The α-cut 0f $\tilde{A}$=(a,b,c,d,e), 0≤α≤1 is $(\alpha)=[A_L(\alpha),A_R(\alpha)]$

Where $A_{L_1}(\alpha)=a+(b-a)\alpha=L_1^{-1}(\alpha)$

$A_{L_2}(\alpha)=b+(c-d)\alpha=L_2^{-1}(\alpha)$

And $A_{R_1}(\alpha)=d-(d-c)\alpha=R_1^{-1}(\alpha)$

$A_{R_2}(\alpha)=e-(e-d)\alpha=R_2^{-1}(\alpha)$

So $L^{-1}(\alpha)=\frac{1}{2}[L_1^{-1}(\alpha)+L_2^{-1}(\alpha)]$

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$=\frac{1}{2}[a+b+(c-a)\alpha]$

$R^{-1}(\alpha)=\frac{1}{2}[R_1^{-1}(\alpha)+R_2^{-1}(\alpha)]$

$=\frac{1}{2}[d+e-(e-c)\alpha]$

Definition2.5: If $\tilde{A}$=(a,b,c,d,e) is a pentagonal fuzzy number then the graded mean integration of

$\tilde{A}$ is defined as,

$$P(\widetilde{A})=\frac{\int_0^{W_A}\left(\frac{L^{-1}(h)+R^{-1}(h)}{2}\right)dh}{\int_0^{W_A}hdh},\ (0\leq h\leq W_A \text{ and } 0\leq W_A\leq 1)$$

$$P(\widetilde{A})=\frac{\int_0^1[\frac{a+b+(c-a)h+d+e-(e-c)h]dh}{2}]}{\int_0^1 hdh}$$

$$=\frac{a+3b+4c+3d+e}{12}$$

III. NOTATION

I (t)=Inventory level at any time, t≥0.

T: Cycle of length.

t_1: Duration of inventory cycle when there is positive inventory.

c_1:Fixed cost.

c_2:Shortages cost per unit.

c_3:Opportunity cost due to lost sales.

c_4:Holding cost per unit, which is variable.

c_5:Unit selling price of the item.

i:Inflation rate per unit time.

r:The discount rate of money.

m:Permissible period of delay in payment.

I_e: Interest earn per unit.

I_p: Interest paid per unit.

TAC(t_1):Toal average cost per unit.

$\tilde{c}_1$=Fuzzy fixed cost.

$\tilde{c}_2$=Fuzzy shortage cost per unit.

$\tilde{c}_3$=Fuzzy opportunity cost due to lost sales.

$\tilde{c}_4$=Fuzzy holding cost, which is variable.

$\tilde{c}_5$=Unit selling price of the item.

$\widetilde{TAC(t_1)}$=Fuzzy total cost per unit.

3.2. ASSUMTION

a: The inventory system involves only one item.

b: The replenishment occur instantaneously at infinite rate.

C: The lead time is negligible.

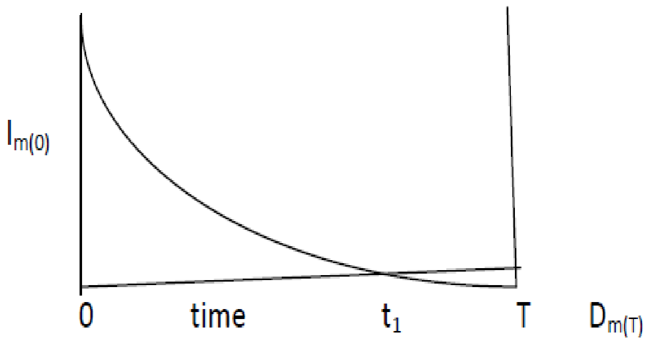
D: Demand rate is power demand, we assume it $d(t/T)^{1/}n$ where d is constant. During the fixed

time T, $\frac{dt^{(1-n)/n}}{nT^{1/n}}$ is demand rate at time t.

E: Backlogging rate is $\frac{1}{1+\delta(T-t)}$, $t_1\leq t\leq T$.

F: Holding cost variable as $e^{\theta t}$, 0≤θ<1.

3.3. MODEL DEVOLOPMENT (CRISP MODEL)



The differential equation describing I(t) as follows

$$\frac{dI(t)}{dt}=-\frac{dt^{\frac{1-n}{n}}}{nT^{1/n}}\quad,\quad 0\leq t\leq t_1 \qquad (3.1)$$

$$\frac{dI(t)}{dt}=-\frac{dt^{\frac{1-n}{n}}}{n\{1+\delta(T-T)\}T^{1/n}},\ t_1\leq t\leq T \qquad (3.2)$$

From (1) we get,

$$I(t)=\frac{d}{T^{1/n}}(t_1^{1/n}-t^{1/n}) \qquad (3.3)$$

From (2) we get,

$$I(t)=-\frac{d}{T^{1/n}}[(t^{1/n}-t_1^{1/n})(1-\delta T)+\frac{\delta}{1+n}\{t^{(1+n)/n}-t_1^{(1+n)/n}\}] \quad (3.4)$$

So the maximum inventory level is,

$$I_m(0)=\frac{d}{T^{1/n}}t_1^{1/n} \qquad (3.5)$$

And the maximum amount of demand backlogging is,

$$D_m(T)=-I(T)=\frac{d}{T^{1/n}}[(T^{1/n}-t_1^{1/n})(1-\delta t)+\frac{\delta}{1+n}\{T^{(1+n)/n}-t_1^{(1+n)/n}\}] \qquad (3.6)$$

Hench the order quantity per cycle is;

$$Q=I_m(0)+D_m(T)=\frac{d}{T^{1/n}}[t_1^{1/n}+(T^{1/n}-t_1^{1/n})\ (1\delta T)+\frac{\delta}{1+n}\{T^{(1+n)/n}-t_1^{(1+n)/n}\}] \qquad (3.7)$$

The fixed cost per cycle is,

$FC=c_1\int_0^T e^{-(r-i)t}dt$, $=c_1\{T+\frac{(i-r)T^2}{2}\}$.

Shortage cost per cycle is,

$Sc=-c_2\int_{t1}^T e^{-(r-i)t}I(t)dt$,

$=-c_2\frac{d}{T^{1/n}}[(1-\delta t)(T-t_1)t_1^{1/n}-\frac{n(1-\delta T)}{n+1}\{T^{(n+1)/n}-t_1^{(n+1)/n}\}+\frac{\delta}{1+n}t_1^{(n+1)/n}(T-T_1)-\frac{n\delta}{(n+1)(2n+1)}\{T^{(2n+1)/n}-t_1^{(2n+1)}$

$-\frac{(1-\delta T)(1-r)}{2}t_1^{1/n}(T^2-T_1^2)-\frac{(1-\delta T)(1-r)n}{2n+1}\{T^{(2n+1)/n}-t_1^{(2n+1)/n}\}+\frac{\delta(i-r)}{2n+2}t_1^{(n+1)/n}(T^2-t_1^2)$

$-\frac{\delta(i-r)n}{(n+1)(3n+1)}\{T^{(3n+1)/n}-t_1^{(3n+1)}\}]$.

Opportunity cost due to lost sales is,

$OC=c_3\int_{t1}^T R(t)[1-\frac{1}{1+\delta(T-t)}]e^{-(r-i)t}dt$

$=c_3\frac{\delta d}{T^{1/n}}[T(T^{1/n}-t_1^{1/n})-\frac{1}{n+1}\{T^{(n+1)/n}-t_1^{(n+1)/n}\}+\frac{(i-r)T}{n+1}\{T^{(n+1)/n}-t_1^{(n+1)/n}\}-\frac{(i-r)}{2n+1}\{T^{(2n+1)/n}-t_1^{(2n+1)/n}\}]$.

Holding cost per cycle is,

As θ, r, and i is too small, so neglecting higher power and product of θ, r, and i.

$HC=c_4\frac{d}{T^{1/n}}\int_0^{t1} e^{\theta t}.e^{-(r-i)t}I(t)dt$

$=c_4\frac{d}{T1/n}[\frac{1}{1+n}t_1^{(n+1)/n}+\frac{(\theta+i-r)}{2+2n}t_1^{(2n+1)/n}]$

CASE-1

When $m\leq t_1$,

Then interest earn per cycle is,

$IE_1=c_5I_e\frac{d}{nT^{1/n}}\int_0^{t1} R(t)e^{-(r-i)t}dt=c_5I_e\frac{d}{T^{1/n}}[\frac{1}{n+1}t_1^{(n+1)/n}+\frac{(i-r)}{(1+2n)}t_1^{(1+2n)/n}]$

And interest pay per cycle is

$IP_1=c_5I_p\frac{d}{T^{1/n}}[\frac{1}{1+n}t_1^{(1+n)/n)}-m(t_1^{1/n}-\frac{n}{n+1}m^{1/n})+(i-r)\{\frac{1}{2+4n}t_1^{(1+2n)/n}-m^2(\frac{1}{2}t_1^{1/n}-\frac{n}{1+2n}m^{1/n})\}]$

Total average cost per cycle is,

$TAC=\frac{1}{T}[FC+SC+OC+HC-IE_1+IP_1]$

$=\frac{d}{T^{(n+1)/n}}[[c_1T^{(n+1)/n}+c_1(i-r)\frac{T^{(2n+1)/n}}{2}-c_2[(1-\delta t)(T-t_1)t_1^{1/n}-\frac{n(1-\delta T)}{n+1}\{T^{(n+1)/n}-t_1^{(n+1)/n}\}+\frac{\delta}{1+n}t_1^{(n+1)/n}(T-T_1)-\frac{n\delta}{(n+1)(2n+1)}\{T^{(2n+1)/n}-t_1^{(2n+1)}-\frac{(1-\delta T)(1-r)}{2}t_1^{1/n}(T^2-T_1^2)-\frac{(1-\delta T)(1-r)n}{2n+1}\{T^{(2n+1)/n}-t_1^{(2n+1)/n}\}+\frac{\delta(i-r)}{2n+2}t_1^{(n+1)/n}(T^2-t_1^2)-\frac{\delta(i-r)n}{(n+1)(3n+1)}\{T^{(3n+1)/n}-t_1^{(3n+1)}\}]+c_3\delta[T(T^{1/n}-t_1^{1/n})-\frac{1}{n+1}\{T^{(n+1)/n}-t_1^{(n+1)/n}\}+\frac{(i-r)T}{n+1}\{T^{(n+1)/n}-t_1^{(n+1)/n}\}-\frac{(i-r)}{2n+1}\{T^{(2n+1)/n}-t_1^{(2n+1)/n}\}]+c_4[\frac{1}{1+n}t_1^{(n+1)/n}+\frac{(\theta+i-r)}{2+4n}t_1^{(2n+1)/n}]$

$-c_5I_e[\frac{1}{n+1}t_1^{(n+1)/n}+\frac{(i-r)}{(1+2n)}t_1^{(1+2n)/n}]+c_5I_p\frac{1}{n}[\frac{1}{1+n}t_1^{(1+n)/n)}-m(t_1^{1/n}-\frac{n}{n+1}m^{1/n})+(i-r)\{\frac{1}{2+4n}t_1^{(1+2n)/n}-m^2(\frac{1}{2}t_1^{1/n}-\frac{n}{1+2n}m^{1/n})\}]]$

For minimum cost it should be,

$$\frac{dTAC(t1)}{dt_1}=0 \text{ and } \frac{d^2TAC(t1)}{dt^2{}_1}>0$$

CASE-2

When $m \geq t_1$

Then interest earn per cycle is,

$$IE_2=c_5I_e\frac{d}{T^{1/n}}[\frac{1}{1+n}t_1^{(1+n)/n}+\frac{(i-r)}{2n+1}t_1^{(2n+1)/n}+(m-t_1)\{t_1^{1/n}+\frac{(i-r)}{(n+1)}t_1^{(n+1)/n}\}]$$

And interest pay per cycle is $IP_2=0$

Total cost per cycle is

$$TAC(t_1)=\frac{1}{T}[FC+SC+OC+HC-IE_2+IP_2)$$

$$=\frac{d}{T^{(n+1)/n}}[[c_1T^{(n+1)/n}+c_1(i-r)\frac{T^{(2n+1)/n}}{2}-c_2\ [(1-\delta t)(T-t_1)t_1^{1/n}-\frac{n(1-\delta T)}{n+1}\{T^{(n+1)/n}-t_1^{(n+1)/n}\}+\frac{\delta}{1+n}t_1^{(n+1)/n}(T-T_1)\ -\frac{n\delta}{(n+1)(2n+1)}\{T^{(2n+1)/n}-t_1^{(2n+1)}\}\ -\frac{(1-\delta T)(1-r)}{2}t_1^{1/n}(T^2-T_1^2)-\frac{(1-\delta T)(1-r)n}{2n+1}\{T^{(2n+1)/n}-t_1^{(2n+1)/n}\}\ +\frac{\delta(i-r)}{2n+2}t_1^{(n+1)/n}(T^2-t_1^2)\ -\frac{\delta(i-r)n}{(n+1)(3n+1)}\{T^{(3n+1)/n}-t_1^{(3n+1)}\}]+c_3\delta\ [T(T^{1/n}-t_1^{1/n})-\frac{1}{n+1}\{T^{(n+1)/n}-t_1^{(n+1)/n}\}+\frac{(i-r)T}{n+1}\{T^{(n+1)/n}-t_1^{(n+1)/n}\}-\frac{(i-r)}{2n+1}\{T^{(2n+1)/n}-t_1^{(2n+1)/n}\}]+c_4\ [\frac{1}{1+n}t_1^{(n+1)/n}+\frac{(\theta+i-r)}{2+4n}t_1^{(2n+1)/n}]\ -c_5I_e\ [\frac{1}{1+n}t_1^{(1+n)/n}+\frac{(i-r)}{2n+1}t_1^{(2n+1)/n}+(m-t_1)\{t_1^{1/n}+\frac{(i-r)}{(n+1)}t_1^{(n+1)/n}\}]]$$

For minimum cost it should be,

$$\frac{dTAC(t1)}{dt_1}=0 \text{ and } \frac{d^2TAC(T1)}{dt^2{}_1}>0$$

3.4. FUZZY MODEL

Due to uncertainly lets us assume that,

$\widetilde{c_1}=(c_1^1,c_1^2,c_1^3,c_1^4,c_1^5)$, $\widetilde{c_2}=(c_2^1,c_2^2,c_2^3,c_2^4,c_2^5)$, $\widetilde{c_3}=(c_3^1,c_3^2,c_3^3,c_3^4,c_3^5)$, $\widetilde{c_4}=(c_4^1,c_4^2,c_4^3,c_4^4,c_4^5)$, $\widetilde{c_5}=c_5^1,c_5^2,c_5^3,c_5^4,c_5^5)$,

be pentagonal fuzzy number then the total average cost is given by, Case-1($m \leq t_1$)

$$t_1)=\frac{d}{T^{(n+1)/n}}[[\hat{c}_1T^{(n+1)/n}+c_1(i-r)\frac{T^{(2n+1)/n}}{2}-\widehat{c_2}\ [(1-\delta t)(T-t_1)t_1^{1/n}-\frac{n(1-\delta T)}{n+1}\{T^{(n+1)/n}-t_1^{(n+1)/n}\}+\frac{\delta}{1+n}t_1^{(n+1)/n}(T-T_1)\ -\frac{n\delta}{(n+1)(2n+1)}\{T^{(2n+1)/n}-t_1^{(2n+1)}\}\ -\frac{(1-\delta T)(1-r)}{2}t_1^{1/n}(T^2-T_1^2)-\frac{(1-\delta T)(1-r)n}{2n+1}\{T^{(2n+1)/n}-t_1^{(2n+1)/n}\}\ +\frac{\delta(i-r)}{2n+2}t_1^{(n+1)/n}(T^2-t_1^2)\ -\frac{\delta(i-r)n}{(n+1)(3n+1)}\{T^{(3n+1)/n}-t_1^{(3n+1)}\}]+\widehat{c_3}\delta\ [T(T^{1/n}-t_1^{1/n})-\frac{1}{n+1}\{T^{(n+1)/n}-t_1^{(n+1)/n}\}+\frac{(i-r)T}{n+1}\{T^{(n+1)/n}-t_1^{(n+1)/n}\}-\frac{(i-r)}{2n+1}\{T^{(2n+1)/n}-t_1^{(2n+1)/n}\}]+\widehat{c_4}\ [\frac{1}{1+n}t_1^{(n+1)/n}+\frac{(\theta+i-r)}{2+4n}t_1^{(2n+1)/n}]-\widehat{c_5}I_e\ [\frac{1}{n+1}t_1^{(n+1)/n}+\frac{(i-r)}{(1+2n)}t_1^{(1+2n)/n}]+\widehat{c_5}I_p\frac{1}{n}\ [\frac{1}{1+n}t_1^{(1+n)/n)}-m(t_1^{1/n}-\frac{n}{n+1}m^{1/n})+(i-r)\{\frac{1}{2+4n}t_1^{(1+2n)/n}-m^2(\frac{1}{2}t_1^{1/n}-\frac{n}{1+2n}m^{1/n})\}]].$$

We defuzzifi the fuzzy total cost $\widetilde{TAC(t_1)}$ by graded mean representation method as follows,

$$\widetilde{TAC(t_1)}=\frac{1}{12}[\widetilde{TAC^1}(t_1), \widetilde{TAC^2}(t_1),\widetilde{TAC^3}(t_1), \widetilde{TAC^4}(t_1), \widetilde{TAC^5}(t_1)]$$

Where $$\widetilde{TAC^r(t_1)}=\frac{d}{T^{(n+1)/n}}[[\widetilde{c^r}_1T^{(n+1)/n}+c_1(i-r)\frac{T^{(2n+1)/n}}{2}-\widetilde{c^r}_2\ [(1-\delta t)(T-t_1)t_1^{1/n}-\frac{n(1-\delta T)}{n+1}\{T^{(n+1)/n}-t_1^{(n+1)/n}\}+\frac{\delta}{1+n}t_1^{(n+1)/n}(T-T_1)-\frac{n\delta}{(n+1)(2n+1)}\{T^{(2n+1)/n}-t_1^{(2n+1)}\}\ -\frac{(1-\delta T)(1-r)}{2}t_1^{1/n}(T^2-T_1^2)\ -\frac{(1-\delta T)(1-r)n}{2n+1}\{T^{(2n+1)/n}-t_1^{(2n+1)/n}\}\ +\frac{\delta(i-r)}{2n+2}t_1^{(n+1)/n}(T^2-t_1^2)\ -\frac{\delta(i-r)n}{(n+1)(3n+1)}\{T^{(3n+1)/n}-t_1^{(3n+1)}\}]+\widetilde{c^r}_3\delta\ [T(T^{1/n}-t_1^{1/n})-\frac{1}{n+1}\{T^{(n+1)/n}-t_1^{(n+1)/n}\}+\frac{(i-r)T}{n+1}\{T^{(n+1)/n}-t_1^{(n+1)/n}\}-\frac{(i-r)}{2n+1}\{T^{(2n+1)/n}-t_1^{(2n+1)/n}\}]+\widetilde{c^r}_4\ [\frac{1}{1+n}t_1^{(n+1)/n}+\frac{(\theta+i-r)}{2+4n}t_1^{(2n+1)/n}]-\widetilde{c^r}_5\ I_e\ [\frac{1}{n+1}t_1^{(n+1)/n}+\frac{(i-r)}{(1+2n)}t_1^{(1+2n)/n}]+\widetilde{c^r}_5I_p\frac{1}{n}\ [\frac{1}{1+n}t_1^{(1+n)/n)}-m(t_1^{1/n}-\frac{n}{n+1}m^{1/n})+(i-r)\{\frac{1}{2+4n}t_1^{(1+2n)/n}-m^2(\frac{1}{2}t_1^{1/n}-\frac{n}{1+2n}m^{1/n})\}]]. \quad r=1,2,3,4,5$$

$$\widetilde{TAC(t_1)}=\frac{1}{12}[\widetilde{TAC_1}(t_1)+3\widetilde{TAC_2}(t_1)+4\widetilde{TAC_3}(t_1)+3\widetilde{TAC_4}(t_1)+\widetilde{TAC_5}(t_1)]$$

For minimum cost it should be,

$$\frac{\widetilde{TAC}(t_1)}{dt_1}=0 \text{ and } \frac{d^2\widetilde{TAC}(t_1)}{dt^2{}_1}>0$$

Case-2($m \geq t_1$)

$$\widetilde{TAC(t_1)} = \frac{d}{T^{(n+1)/n}}[[\widetilde{c_1}T^{(n+1)/n}+c_1(i-r)\frac{T^{(2n+1)/n}}{2} - \widetilde{c_2}\ [(1-\delta t)(T-t_1)t_1^{1/n} - \frac{n(1-\delta T)}{n+1}\{T^{(n+1)/n}-t_1^{(n+1)/n}\}+\frac{\delta}{1+n}t_1^{(n+1)/n}(T-T_1) - \frac{n\delta}{(n+1)(2n+1)}\{T^{(2n+1)/n}-t_1^{(2n+1)} - \frac{(1-\delta T)(1-r)}{2}t_1^{1/n}(T^2-T_1^2) - \frac{(1-\delta T)(1-r)n}{2n+1}\{T^{(2n+1)/n}-t_1^{(2n+1)/n}\} + \frac{\delta(i-r)}{2n+2}t_1^{(n+1)/n}(T^2-t_1^2) - \frac{\delta(i-r)n}{(n+1)(3n+1)}\{T^{(3n+1)/n}-t_1^{(3n+1)}\}] + \widetilde{c_3}\delta\ [T(T^{1/n}-t_1^{1/n}) - \frac{1}{n+1}\{T^{(n+1)/n}-t_1^{(n+1)/n}\}+\frac{(i-r)T}{n+1}\{T^{(n+1)/n}-t_1^{(n+1)/n}\} - \frac{(i-r)}{2n+1}\{T^{(2n+1)/n}-t_1^{(2n+1)/n}\}] + \widetilde{c_4}\ [\frac{1}{1+n}t_1^{(n+1)/n}+\frac{(\theta+i-r)}{2+4n}t_1^{(2n+1)/n}] - \widetilde{c_5}I_e\ [\frac{1}{1+n}t_1^{(1+n)/n}+\frac{(i-r)}{2n+1}t_1^{(2n+1)/n}+(m-t_1)\{t_1^{1/n}+\frac{(i-r)}{(n+1)}t_1^{(n+1)/n}\}]]$$

We defuzzifi the fuzzy total cost $\widetilde{TAC}$ by graded mean representation method as follows,

$$\widetilde{TAC(t_1)}=\frac{1}{12}[\widetilde{TAC}^1(t_1), \widetilde{TAC}^2(t_1), \widetilde{TAC}^3\ (t_1), \widetilde{TAC}^4(t_1), \widetilde{TAC}^5(t_1)]$$

Where
$$\widetilde{TAC^r}(t_1) = \frac{d}{T^{(n+1)/n}}[[\widetilde{c^r}_1T^{(n+1)/n}+\widetilde{c^r}_1(i-r)\frac{T^{(2n+1)/n}}{2} - \widetilde{c^r}_2[(1-\delta t)(T-t_1)t_1^{1/n} - \frac{n(1-\delta T)}{n+1}\{T^{(n+1)/n}-t_1^{(n+1)/n}\}+\frac{\delta}{1+n}t_1^{(n+1)/n}(T-T_1) - \frac{n\delta}{(n+1)(2n+1)}\{T^{(2n+1)/n}-t_1^{(2n+1)} - \frac{(1-\delta T)(1-r)}{2}t_1^{1/n}(T^2-T_1^2) - \frac{(1-\delta T)(1-r)n}{2n+1}\{T^{(2n+1)/n}-t_1^{(2n+1)/n}\} + \frac{\delta(i-r)}{2n+2}t_1^{(n+1)/n}(T^2-t_1^2) - \frac{\delta(i-r)n}{(n+1)(3n+1)}\{T^{(3n+1)/n}-t_1^{(3n+1)}\}] + \widetilde{c^r}_3\delta\ [T(T^{1/n}-t_1^{1/n}) - \frac{1}{n+1}\{T^{(n+1)/n}-t_1^{(n+1)/n}\}+\frac{(i-r)T}{n+1}\{T^{(n+1)/n}-t_1^{(n+1)/n}\} - \frac{(i-r)}{2n+1}\{T^{(2n+1)/n}-t_1^{(2n+1)/n}\}]+\widetilde{c^r}_4\ [\frac{1}{1+n}t_1^{(n+1)/n}+\frac{(\theta+i-r)}{2+4n}t_1^{(2n+1)/n}] - \widetilde{c^r}_5\ I_e\ [\frac{1}{1+n}t_1^{(1+n)/n}+\frac{(i-r)}{2n+1}t_1^{(2n+1)/n}+(m-t_1)\{t_1^{1/n}+\frac{(i-r)}{(n+1)}t_1^{(n+1)/n}\}]].\quad r=1,2,3,4,5$$

$$\widetilde{TAC(t_1)} = \frac{1}{12}[\widetilde{TAC_1} + (t_1)+3\widetilde{TAC_2}\ (t_1)+ 4\widetilde{TAC_3}\ (t_1)+3\widetilde{TAC_4}\ (t_1) +\widetilde{TAC_5}\ (t_1)]$$

For minimum cost it should be,

$$\frac{\widetilde{TAC}(t_1)}{dt_1}=0 \text{ and } \frac{d^2\widetilde{TAC}(t_1)}{dt^2{}_1}>0$$

IV. NUMERICAL SOLUTION

CASE-1($m \leq t_1$) for crisp model: Let us take the in-put value:

C_1	C_2	C_3	C_4	C_5	i	r	δ	θ	I_e	I_p	m	n	d	T
100	5	6	5	2	0.1	0.2	0.1	0.2	0.1	0.2	0.1	3	20	1

And the out-put value:

Q	t_1	TAC(t_1)
19.532	0.155	1847.694

For fuzzy model:

$\widetilde{c_1}$=(90,95,100,105,110), $\widetilde{c_2}$ =(3,4,5,6,7), $\widetilde{c_3}$=(4,5,6,7,8),

$\widetilde{c_4}$ =(3,4,5,6,7), $\widetilde{c_5}$=(0,1,2,3,4)

The solution of fuzzy model by graded mean representation is,

1. When $\widetilde{c_1},\widetilde{c_2},\widetilde{c_3},\widetilde{c_4},\widetilde{c_5}$ are all pentagonal fuzzy numbers then,
   TAC (t_1) =1866.324, t_1=0.142
2. When $\widetilde{c_1},\widetilde{c_2},\widetilde{c_3},\widetilde{c_4}$, are all pentagonal fuzzy numbers then
   TAC (t_1) =1864.691, t_1=0.150
3. When $\widetilde{c_1},\widetilde{c_2},\widetilde{c_3}$,are pentagonal fuzzy numbers then,
   TAC (t_1) =1861.000, t_1=0.133
4. When $\widetilde{c_1},\widetilde{c_2}$,are pentagonal fuzzy numbers then,
   TAC (t_1) =1861.861, t_1=0.137
5. When $\widetilde{c_1}$, are pentagonal fuzzy numbers then,
   TAC (t_1) =1847.676, t_1=.155

CASE-2($m \geq t_1$)

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For crisp model: Let us take the in-put value:

C_1	C_2	C_3	C_4	C_5	i	r	δ	θ	I_e	I_p	m	n	d	T
100	5	6	5	2	0.1	0.2	0.1	0.2	0.1	0.2	0.2	3	20	1

And the out-put value:

Q	t_1	$TAC(t_1)$
19.516	0.147	1850.552

For fuzzy model

$\tilde{c}_1$ =(90,95,100,105,110), $\tilde{c}_2$ =(22,23,24,25,26), $\tilde{c}_3$=(10,11,12,13,14)

$\tilde{c}_4$ =(15,16,17,18,19), $\tilde{c}_5$=(5,6,7,8,9)

The solution of fuzzy model by graded mean representation is,

(1) When $\tilde{c}_1,\tilde{c}_2,\tilde{c}_3,\tilde{c}_4,\tilde{c}_5$ are all pentagonal fuzzy numbers then,
$TAC(t_1)$=1843.779, t_1=0.142

(2) When $\tilde{c}_1,\tilde{c}_2,\tilde{c}_3,\tilde{c}_4$,are pentagonal fuzzy numbers then,
$TAC(t_1)$=1864.030, t_1=0.143

(3) When $\tilde{c}_1,\tilde{c}_2,\tilde{c}_3$, are pentagonal fuzzy numbers then,

$TAC(t_1)$=1863.710, t_1=0.127

(4) When $\tilde{c}_1,\tilde{c}_2$,are pentagonal fuzzy numbers then,

$TAC(t_1)$=1864.599, t_1=0.131

(5) When $\tilde{c}_1$ are pentagonal fuzzy numbers then,

$TAC(t_1)$=1850.152, t_1=0.147

V. SENSITIVITY ANALYSIS:

Case-1 ($m \le t_1$)

We now examine to sensitivity analysis of the optimal solution of the model for change in I, keeping the other parameters unchanged. The initial data from the above numerical example.

I	% change	t_1	$TAC(t_1)$
0.05	-50	0.173	1797.097
0.06	-40	0.169	1807.242
0.07	-30	0.165	1817.373
0.08	-20	0.161	1827.491
0.09	-10	0.158	1837.591
0.10	0	0.155	1847.694
0.11	10	0 .152	1857.781
0.12	20	0.149	1867.860
0.13	30	0.147	1877.931
0.14	40	0.144	1887.996
0.15	50	0.142	1898.054

5.2. (a) Effect, for increment parameter i,

(a) The inventory time (t_1) decrease.
(b) Total average cost (TAC) increase.

Case-2($m \geq t_1$)

We now examine to sensitivity analysis of the optimal solution of the model for change in I, keeping the other parameters unchanged. The initial data from the above numerical example.

i	% change	t_1	TAC(t_1)
0.05	-50	0.163	1792.057
0.06	-40	0.159	1803.232
0.07	-30	0.156	1814.353
0.08	-20	0.153	1825.461
0.09	-10	0.150	1837.527
0.10	0	0.155	1847.694
0.11	10	0 .147	1859.781
0.12	20	0.143	1871.860
0.13	30	0.140	1881.931
0.14	40	0.137	1892.996
0.15	50	0.135	1903.054

5.2. (b).Effect, for increment parameter i,

(a) The inventory time(t_1) decrease.
(b) Total average cost(TAC) increase

VI. CONCLUSON

In this paper, we have proposed a real life inventory problem in a fuzzy environment and presented solution along with sensitivity analysis approach. The inventory model developed with power pattern demand with shortages. Shortages have been allow partially backlogged in this model. In case where large portion of demand occurs at the beginning of the period the author, use n>1 and when it is large at end we use 0<n<1. This model has been developed for single item. In this paper, we have considered pentagonal fuzzy number and solved by graded mean integration method. In future, the other type of membership functions such as piecewise linear hyperbolic, L-R fuzzy number etc can be considered to construct the membership function and then model can be easily solved.

REFERENCE

1. Bellman, R.E, And Zadeh (1970), Decision making in a fuzzy environment, Management Science 17, B141-B164
2. Carlsson, c. And p. Korhonen (1986), A parametric approach to fuzzy linear programming, Fuzzy sets and systems, 17-30.
3. CLARK, A.J, (1992), An informal survey of multy-echelon inventory theory , naval research logistics Quarterly 19, 621-650.
4. Dutta, d.j.r. Rao, and r.n tiwary (1993), Effect of tolerance in fuzzy linear fractional programming, fuzzy sets and systems 55, 133-142.
5. Hamacher, H.Leberling And H.J.Zimmermann (1978), Sensitivity Analysis in fuzzy linear Programming Fuzzy sets and systems 1, 269-281.
6. Hadley, g. And t.m. White (1963),Analysis of inventory system, Prentice-Hall, Englewood Cliffs, NJ.
7. Khun, h.w and a.w. Tucker (1951), Non-linear programming, proceeding second Berkeley symposium Mathematical and probability (ed) Nyman ,J.University of California press 481-492.
8. Raymond, F.E (1931), Quantity and Economic in manufacturing, Mcgraw-Hill, New York.
9. Zadeh, L.A (1965), Fuzzy sets, Information and Control, 8, 338-353.
10. Zimmermann, H.J.(1985),Application of fuzzy set theory to mathematical programming, Information Science, 36, 29-58.