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A Two-Warehouse Inventory Model for Decaying Items with Exponential Demand and Variable Holding Cost

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Abstract- This chapter presents a two warehouses inventory model for deteriorating items. It is assumed that the inventory costs (including holding cost and deterioration cost) in RW are higher than those in OW. Demand is taken exponentially increasing with time. Holding cost is taken as variable and it is linear increasing function of time. Shortages are allowed in the owned warehouse and the backlogging rate of unsatisfied demand is assumed to be a decreasing function of the waiting time. Profit maximization technique is used in this study.

Keywords- OW. RW.

I. INTRODUCTION

Some inventory models were formulated in a static environment where the demand is assumed to be constant and steady over a finite planning horizon. In fact, the constant demand assumption is only valid during the maturity phase of time. Many items of inventory such as electronic products, fashionable clothes, tasty food products and domestic goods generate increasing sales after gaining consumer's acceptance. The sales for the other products may decline drastically due to the introduction of more competitive products or due to the change of consumer's preferences. Therefore the demand of the product during its growth and decline phases can be well approximated by continuous-time-dependent function such as linear or exponential.

Donaldson (1977) developed an optimal algorithm for solving classical no-shortage inventory model analytically with linear trend in demand over fixed time horizon. Dave, U. (1989) proposed a deterministic lot-size inventory model with shortages and a linear trend in demand. Goswami and Chaudhuri (1991) discussed different types of inventory models with linear trend in demand. Aggarwal and Jaggi (1995) developed ordering policies of deteriorating items under permissible delay in payments. The demand and deterioration were consumed as constant. Hariga (1995) studied the effects of inflation and time value of money on an inventory model with time-dependent demand rate and shortages. Mandal and Maiti (1999) discussed an inventory of damageable items with variable replenishment rate and deterministic demand. Balkhi and Benkherouf (2004) developed an inventory model for deteriorating items with stock dependent and time varying demand rates over a finite planning horizon. Mahapatra. N. K. and Maiti, M. (2005) presented the multi objective and single objective inventory models of stochastically deteriorating items are developed in which demand is a function of inventory level and selling price of the commodity. Wu et al. (2006) proposed an optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. Panda et al. (2007) considered an EOQ model with ramp-type demand and Weibull distribution deterioration. Sana and Chaudhuri (2008) formulated the retailer's profit maximizing strategy. Increasing deterministic demands were also discussed.

A deterministic inventory model for deteriorating items with two warehouses is developed. In this chapter it is assumed that the inventory costs (including holding cost and deterioration cost) in RW are higher than those in OW. Demand is taken exponentially increasing with time. Holding cost is taken as variable and it is linear increasing function of time. In addition, shortages are allowed in the owned warehouse and the backlogging rate of unsatisfied demand is assumed to be a decreasing function of the waiting time. Profit maximization technique is used in this study.

II. ASSUMPTIONS AND NOTATIONS

The mathematical model of the two-warehouse inventory problem is based on the following notations and assumptions:

Notations:

- C the purchase cost per unit.
- C' inventory ordering (or replenishment) cost per order.
- w the capacity of the owned warehouse.
- p the selling price per unit time, where $p > C$.
- Q the ordering quantity.
- S the maximum inventory level per cycle.
- $h_1 = a_1 + b_1 t$ the holding cost per unit per unit time in OW.
- $h_2 = a_2 + b_2 t$ the holding cost per unit per unit time in RW, $h_2 > h_1$.
- C_s the backlogging cost per unit per unit time.
- L the opportunity cost (i.e. goodwill cost) per unit.
- T the length of replenishment cycle.
- t_1 the time at which the inventory level reaches to zero in RW.
- t_2 the time at which the inventory level reaches to zero in OW.
- $I_1(t)$ the level of the inventory in RW at time t.
- $I_2(t)$ the level of the inventory in OW at time t.

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$I_3(t)$ the level of the negative inventory at time t.

α the deterioration rate in OW, where $0 \le \alpha < 1$.

β the deterioration rate in RW, where $0 \le \beta < 1$.

Assumptions:

1. Replenishment rate is infinite and Lead time is zero.
2. Holding cost varies with time.
3. The time horizon of the inventory system is infinite.
4. The goods of OW are consumed only after consuming the goods kept in RW.
5. The owned warehouse has a fixed capacity of w units and the rented warehouse has unlimited capacity.
6. $D = Ae^{\lambda t}$ is the demand rate per unit time, where A is the initial demand and $\lambda > 0$.
7. For deteriorating items a fraction of the on-hand inventory deteriorates per unit of time, in both the warehouses with different rates.
8. Shortages are allowed and unsatisfied demand is backlogged at a rate $e^{-\sigma x}$, where x is the waiting time and backlogging parameter σ is a positive constant.
9. The unit inventory cost (including holding cost and deterioration cost) per unit time in RW are higher than those in OW, i.e. $h_2 + \beta C > h_1 + \alpha C$.

III. FORMULATION AND SOLUTION OF THE MODEL

For a traditional model, at the time t = 0, a lot size of certain units of a product enters in the system from which a portion is used to meet the partially backlogged items towards pervious shortages and w units of remaining items are kept in OW and rest is stored in RW. During the interval $(0,t_1)$ the inventory in RW gradually decreases due to demand and deterioration and it vanishes at $t = t_1$. In OW, the inventory w decreases during $(0,t_1)$ only due to deterioration, but in (t_1, t_2) inventory depleted due to the both demand and deterioration. At time $t = t_2$, both the warehouses are empty and there after shortages occurs. At the time T, partially backordered quantity is supplied to the customer at the beginning of the next cycle. At the time T, the replenishment cycle restarts. The objective to find out the maximum total average profit by considering the relevant cost (including ordering, holding, deterioration, backlogging and lost sale) per unit time of the inventory system.

Inventory level

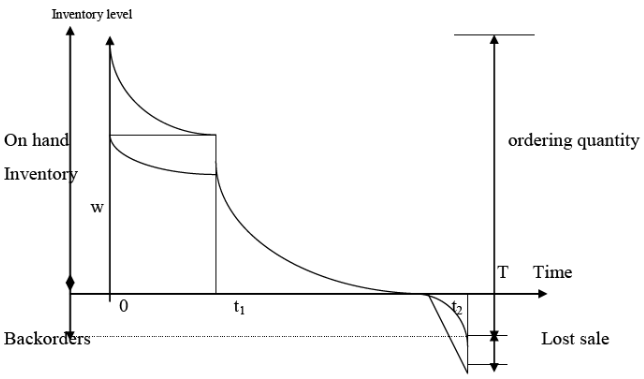

Fig.1 Graphical representation of a two-warehouse inventory system

The inventory levels in RW and OW are given by the following differential equations:

$$\frac{dI_1(t)}{dt} + \beta I_1(t) = -Ae^{\lambda t}, \qquad 0 \le t \le t_1. \qquad \dots (6.1)$$

With the condition $I_1(t_1) = 0$ and

$$\frac{dI_2(t)}{dt} + \alpha I_2(t) = 0, \quad 0 \le t \le t_1. \qquad \dots (6.2)$$

With the condition $I_2(0) = w$

The solution of equation (6.1) and (6.2) are respectively given by as follows:

$$\Rightarrow I_1(t) = \frac{A}{(\beta+\lambda)}\left[e^{(\beta+\lambda)t_1 - \beta t} - e^{\lambda t}\right], \qquad \dots (6.3)$$

$$I_2(t) = we^{-\alpha t}, \quad 0 \le t \le t_1. \qquad \dots (6.4)$$

Again, in the interval (t_1, t_2), the inventory in OW is depicted due to the joint effect of the demand and deterioration. The inventory level in OW is given by the following differential equation:

$$\frac{dI_2(t)}{dt} + \alpha I_2(t) = -Ae^{\lambda t}, \qquad t_1 \le t \le t_2. \qquad \dots (6.5)$$

With the condition $I_2(t_2) = 0$

The solution of equation (6.5) is similarly given by

$$I_2(t) = \frac{A}{(\lambda+\alpha)}\left[e^{(\lambda+\alpha)t_2 - \alpha t} - e^{\lambda t}\right], \quad t_1 \le t \le t_2. \qquad \dots (6.6)$$

From equations (6.4) and (6.6) and due to the continuity at $t = t_1$, We get

$$I_2(t_1) = We^{-\alpha t_1} = \frac{A}{(\lambda+\alpha)}[e^{(\lambda+\alpha)t_2 - \alpha t_1} - e^{\lambda t_1}]$$

$$\Rightarrow W = \frac{A}{(\lambda+\alpha)}[e^{(\lambda+\alpha)t_2} - e^{(\lambda+\alpha)t_1}]$$

$$\Rightarrow W = \frac{Ae^{(\lambda+\alpha)t_1}}{(\lambda+\alpha)}\left[e^{(\lambda+\alpha)(t_2 - t_1)} - 1\right]$$

$$\Rightarrow \frac{W}{A}(\lambda+\alpha)e^{-(\lambda+\alpha)t_1} + 1 = e^{(\lambda+\alpha)(t_2 - t_1)}$$

$$\Rightarrow t_2 = t_1 + \frac{1}{(\lambda+\alpha)}\ln\left(\frac{W}{A}(\lambda+\alpha)e^{-(\lambda+\alpha)t_1} + 1\right)$$

$$\Rightarrow t_2 = t_1 + \frac{1}{(\lambda+\alpha)}\left[\ln\left(\frac{W}{A}(\lambda+\alpha) + e^{(\lambda+\alpha)t_1}\right) - (\lambda+\alpha)t_1\right]$$

$$= t_1 + \frac{1}{(\lambda+\alpha)}\ln\left(\frac{W}{A}(\lambda+\alpha) + e^{(\lambda+\alpha)t_1}\right) - t_1$$

$$\therefore t_2 = \frac{1}{(\lambda+\alpha)}\ln\left(\frac{W}{A}(\lambda+\alpha) + e^{(\lambda+\alpha)t_1}\right) \qquad \dots (6.7)$$

Equation (6.7) shows that t_2 is a function of t_1. Then taking the first order derivative of t_2 with respect to t_1,

We get

$$\frac{dt_2}{dt_1} = \frac{e^{(\lambda+\alpha)t_1}}{\left[\frac{W(\lambda+\alpha)}{A} + e^{(\lambda+\alpha)t_1}\right]} \qquad \dots (6.8)$$

$$\frac{dt_2}{dt_1} = \frac{1}{\left[\frac{W(\lambda+\alpha)}{A}e^{-(\lambda+\alpha)t_1} + 1\right]} < 1$$

$$\Rightarrow \frac{dt_2}{dt_1} - 1 < 0 \quad \text{holds}$$

Furthermore, at time t_2 the inventory level becomes zero in OW and shortage occurs. In the interval (t_2, T) the inventory level depends on the demand which is partially backlogged. The inventory level is given by the following differential equation:

$$\frac{dI_3(t)}{dt} = -Ae^{\lambda t}e^{-\sigma t}, \quad t_2 \le t \le T. \quad \dots (6.9)$$

With the condition $I_3(t_2) = 0$

The solution of equation (6.9) is given by

$$I_3(t) = \frac{A}{(\lambda-\sigma)}\left[e^{(\lambda-\sigma)t_2} - e^{(\lambda-\sigma)t}\right], \quad t_2 \le t \le T. \quad \dots (6.10)$$

The order quantity over the replenishment cycle can be determined by

$$Q = I_1(0) + I_2(0) - I_3(T)$$

$$= \frac{A}{(\lambda+\beta)}\left[e^{(\lambda+\beta)t_1} - 1\right] + W - \frac{A}{(\lambda-\sigma)}\left[e^{(\lambda-\sigma)t_2} - e^{(\lambda-\sigma)T}\right] \quad \dots (6.11)$$

The maximum inventory level per cycle is given by

$$S = I_1(0) + I_2(0)$$

$$= \frac{A}{(\lambda+\beta)}\left[e^{(\lambda+\beta)t_1} - 1\right] + W \quad \dots (6.12)$$

Now the total profit per cycle consists of following elements-

Ordering cost = C' … (6.13)

Holding cost per cycle in RW

$$HC_1 = \int_0^{t_1} h_2 I_1(t)dt$$

$$= \int_0^{t_1} (a_2 + b_2 t)\frac{A}{(\beta+\lambda)}\left[e^{(\beta+\lambda)t_1 - \beta t} - e^{\lambda t}\right]dt$$

$$= \frac{Ae^{\lambda t_1}}{(\beta+\lambda)}\left[\frac{1}{\beta^2}(\beta a_2 + b_2)\left(e^{\beta t_1} - 1\right) - \frac{1}{\lambda^2}(\lambda a_2 - b_2)\left(1 - e^{-\lambda t_1}\right) - \frac{(\beta+\lambda)}{\lambda\beta}b_2 t_1\right] \quad .. (6.14)$$

Holding cost per cycle in OW

$$HC_2 = \left[\int_0^{t_1} h_1 I_2(t)dt + \int_{t_1}^{t_2} h_1 I_2(t)dt\right]$$

$$= \int_0^{t_1}(a_1 + b_1 t)we^{-\alpha t}dt + \int_{t_1}^{t_2}(a_1 + b_1 t)\frac{A}{(\lambda+\alpha)}\left[e^{(\lambda+\alpha)t_2 - \alpha t} - e^{\lambda t}\right]dt$$

$$= \frac{w}{\alpha^2}\left[(\alpha a_1 + b_1)(1 - e^{-\alpha t_1}) - \alpha b_1 t_1 e^{-\alpha t_1}\right] - \frac{Ae^{\lambda t_2}}{\lambda^2\alpha^2}\left\{(a_1 + b_1 t_2)\lambda\alpha + b_1(\lambda-\alpha)\right\}$$

$$+ \frac{Ae^{\lambda t_1}}{(\lambda+\alpha)\lambda^2\alpha^2}\left[(a_1 + b_1 t_1)\lambda\alpha\left(\alpha + \lambda e^{(\lambda+\alpha)(t_2 - t_1)}\right) - b_1\left(\alpha^2 - \lambda^2 e^{(\lambda+\alpha)(t_2 - t_1)}\right)\right] \quad \dots (6.15)$$

The total holding cost per cycle in RW and OW

$$HC = HC_1 + HC_2$$

$$= \frac{Ae^{\lambda t_1}}{(\beta+\lambda)}\left[\frac{1}{\beta^2}(\beta a_2 + b_2)\left(e^{\beta t_1} - 1\right) - \frac{1}{\lambda^2}(\lambda a_2 - b_2)\left(1 - e^{-\lambda t_1}\right)\right]$$

$$- \frac{Ae^{\lambda t_1}}{\lambda\beta}b_2 t_1 + \frac{w}{\alpha^2}\left[(\alpha a_1 + b_1)(1 - e^{-\alpha t_1}) - \alpha b_1 t_1 e^{-\alpha t_1}\right]$$

$$- \frac{A}{\lambda^2\alpha^2}\left[e^{\lambda t_2}\left\{(a_1 + b_1 t_2)\lambda\alpha + b_1(\lambda-\alpha)\right\} - \frac{e^{\lambda t_1}}{(\lambda+\alpha)}\left\{b_1\left(\alpha^2 - \lambda^2 e^{(\lambda+\alpha)(t_2 - t_1)}\right)\right.\right.$$

$$\left.\left.-(a_1 + b_1 t_1)\lambda\alpha\left(\alpha + \lambda e^{(\lambda+\alpha)(t_2 - t_1)}\right)\right\}\right] \quad \dots (6.16)$$

Backlogging cost per cycle

$$SC = -C_s\int_{t_2}^{T} I_3(t)dt$$

$$= -C_s\int_{t_2}^{T}\frac{A}{(\lambda-\sigma)}\left[e^{(\lambda-\sigma)t_2} - e^{(\lambda-\sigma)t}\right]dt$$

$$= \frac{-C_s A}{(\lambda-\sigma)^2}e^{(\lambda-\sigma)t_2}\left[(T - t_2)(\lambda-\sigma) - e^{(\lambda-\sigma)(T - t_2)} + 1\right] \quad \dots (6.17)$$

Opportunity cost due to lost sales per cycle

$$OC = L\int_{t_2}^{T}\left(1 - e^{-\sigma t}\right)Ae^{\lambda t}dt$$

$$= LA\int_{t_2}^{T}\left(e^{\lambda t} - e^{(\lambda-\sigma)t}\right)dt$$

$$= LA\left[\frac{1}{\lambda}\left(e^{\lambda T} - e^{\lambda t_2}\right) - \frac{1}{(\lambda-\sigma)}\left(e^{(\lambda-\sigma)T} - e^{(\lambda-\sigma)t_2}\right)\right] \dots (6.18)$$

Purchase cost per cycle

$$PC = CQ$$

$$= C\left[\frac{A}{(\lambda+\beta)}\left[e^{(\lambda+\beta)t_1} - 1\right] + W + \frac{A}{(\lambda-\sigma)}\left[e^{(\lambda-\sigma)T} - e^{(\lambda-\sigma)t_2}\right]\right] \quad \dots (6.19)$$

Sales revenue per cycle

$$SR = p\left[\int_0^{t_2} Ae^{\lambda t}dt + \int_{t_2}^{T} Ae^{\lambda t}e^{-\sigma t}dt\right]$$

$$= pA\left[\frac{1}{\lambda}\left(e^{\lambda t_2} - 1\right) + \frac{1}{(\lambda-\sigma)}\left(e^{(\lambda-\sigma)T} - e^{(\lambda-\sigma)t_2}\right)\right]$$

$$= \frac{pA}{\lambda(\lambda-\sigma)}\left[(\lambda-\sigma)\left(e^{\lambda t_2} - 1\right) + \lambda\left(e^{(\lambda-\sigma)T} - e^{(\lambda-\sigma)t_2}\right)\right] \dots 6.20)$$

The total profit per unit is given by

$P(t_1, T) = \frac{1}{T}$ [Sales Revenue – Ordering cost – Holding cost – Backlogging cost – opportunity cost – Purchase cost]

$$= \frac{1}{T}\left[SR - C' - HC - SC - OC - PC\right]$$

$$P(t_1,T) = \frac{1}{T}\left[\frac{pA}{\lambda(\lambda-\sigma)}\left[(\lambda-\sigma)\left(e^{\lambda t_2} - 1\right) + \lambda\left(e^{(\lambda-\sigma)T} - e^{(\lambda-\sigma)t_2}\right)\right] - C'\right.$$

$$- \frac{Ae^{\lambda t_1}}{(\beta+\lambda)}\left[\frac{1}{\beta^2}(\beta a_2 + b_2)\left(e^{\beta t_1} - 1\right) - \frac{1}{\lambda^2}(\lambda a_2 - b_2)\left(1 - e^{-\lambda t_1}\right)\right]$$

$$+ \frac{Ae^{\lambda t_1}}{\lambda\beta}b_2 t_1 - \frac{w}{\alpha^2}\left[(\alpha a_1 + b_1)(1 - e^{-\alpha t_1}) - \alpha b_1 t_1 e^{-\alpha t_1}\right]$$

$$+ \frac{A}{\lambda^2\alpha^2}\left[e^{\lambda t_2}\left\{(a_1 + b_1 t_2)\lambda\alpha + b_1(\lambda-\alpha)\right\} - \frac{e^{\lambda t_1}}{(\lambda+\alpha)}\left\{b_1\left(\alpha^2 - \lambda^2 e^{(\lambda+\alpha)(t_2 - t_1)}\right)\right.\right.$$

$$\left.\left.-(a_1 + b_1 t_1)\lambda\alpha\left(\alpha + \lambda e^{(\lambda+\alpha)(t_2 - t_1)}\right)\right\}\right] +$$

$$\frac{C_s A}{(\lambda-\sigma)^2}e^{(\lambda-\sigma)t_2}\left[(T - t_2)(\lambda-\sigma) - e^{(\lambda-\sigma)(T - t_2)} + 1\right]$$

$$-LA\left[\frac{1}{\lambda}\left(e^{\lambda T} - e^{\lambda t_2}\right) - \frac{1}{(\lambda-\sigma)}\left(e^{(\lambda-\sigma)T} - e^{(\lambda-\sigma)t_2}\right)\right]$$

$$\left.-C\left[\frac{A}{(\lambda+\beta)}\left\{e^{(\lambda+\beta)t_1} - 1\right\} + W + \frac{A}{(\lambda-\sigma)}\left\{e^{(\lambda-\sigma)T} - e^{(\lambda-\sigma)t_2}\right\}\right]\right] \quad \dots (6.21)$$

To maximize the total profit per unit time, the optimal values of t_1 and T can be obtained by solving the following equations simultaneously.

$$\frac{\partial P(t_1,T)}{\partial t_2} = 0 \quad \text{and} \quad \frac{\partial P(t_1,T)}{\partial T} = 0 \quad \dots (6.22)$$

Provided, they satisfy the following conditions

$$\frac{\partial^2 P(t_1,T)}{\partial t_2^2} > 0, \quad \frac{\partial^2 P(t_1,T)}{\partial T^2} > 0 \quad \text{And}$$

$$\left(\frac{\partial^2 P(t_1,T)}{\partial t_2^2}\right)\left(\frac{\partial^2 P(t_1,T)}{\partial T^2}\right)-\left(\frac{\partial^2 P(t_1,T)}{\partial t_2 \partial T}\right)^2 < 0 \quad \dots (6.23)$$

IV. NUMERICAL EXAMPLE

Example 1: With Shortage Case:

A=50, λ=0.3, W=100, α=0.05,
β=0.03, σ=-0.6t, $\delta(t)=e^{-0.6t}$, C'=90,
C=10, h_1=1.0, h_2=2.5, c_s=3,
L=14 in appropriate units. Then Total Profit=352.267

Example 2: Without Shortage Case:

A=50, λ=0.3, W=100, α=0.05,
β=0.03, σ=-0.6t, $\delta(t)=e^{-0.6t}$, C'=90,
C=10, h_1=1.0, h_2=2.5 in appropriate units. Then Total Profit=244.7094

V. SENSITIVITY ANALYSIS

To study the effects of changes of the parameters on the optimal profit derived by proposed method, a sensitivity analysis is performed considering the numerical examples given above. Sensitivity analysis is performed by changing (increasing or decreasing) the parameters by 20% and 50% and taking one parameter at a time, keeping the remaining parameters at their original values. The results are shown in table 1 and table 2 for with shortage case and without shortage case respectively.

Table 1.1 Sensitivity Analysis with respect to the various parameters with shortages

Parameter	% Change in Parameter	% Change in profit	% Change in p	% Change in Q	% Change in T	% Change in t_1
A	-50	2.4365	-0.7643	-7.5437	-9.7654	-9.7654
	-20	0.4524	-0.3456	-16.987	-6.6543	-6.6543
	20	-0.6525	0.0536	6.9864	4.8765	4.8765
	50	-1.9345	0.0874	19.775	8.9876	8.9876
A	-50	-89.5096	-46.9753	-53.7543	59.5336	59.5336
	-20	-49.6578	-24.8674	-21.7754	12.7643	12.7643
	20	56.0358	35.8757	28.6543	-8.6538	-8.6538
	50	97.7655	56.7543	45.8665	-18.4667	-18.4667
Σ	-50	0.4388	-0.0876	8.9869	9.4322	9.4322
	-20	0.1654	-0.0324	3.9875	3.8753	3.8753
	20	-0.1365	0.0215	-2.6548	-2.7542	-2.7542
	50	-0.8755	0.0975	-6.6542	-6.8642	-6.8642
h_1	-50	0.6532	-0.1356	8.3457	7.4567	7.4567
	-20	0.5327	-0.0843	4.8765	2.9875	2.9875
	20	-0.0765	0.0433	-3.4378	-2.8655	-2.8655
	50	-0.3468	0.2234	-7.2467	-6.9854	-6.9854
h_2	-50	0.6437	-0.5634	7.2568	8.9786	8.9786
	-20	0.3457	-0.2457	3.8068	4.8653	4.8653
	20	-0.1254	0.1675	-2.5689	-3.9753	-3.9753
	50	-0.0569	0.5367	-8.4575	9.4456	9.4456
A	-50	0.8448	-0.0864	8.9964	9.6548	9.6548
	-20	0.7636	-0.9974	3.9865	3.9875	3.9875
	20	-0.8644	0.3434	-2.9854	-2.9876	-2.9876
	50	-0.8712	0.7643	-6.2134	-6.9854	-6.9854
B	-50	0.5642	-0.7643	8.5436	8.0875	8.0875
	-20	0.0743	-0.4345	3.9875	3.9754	3.9754
	20	-0.9083	0.9764	-2.0987	-2.4563	-2.4563
	50	-0.6367	0.9875	-2.7643	-6.7653	-6.7653
C	-50	28.7644	-8.9744	19.5677	5.3731	5.3731
	-20	12.8767	-3.3456	6.7543	1.7654	1.7654
	20	-10.7657	3.8644	-6.9763	-1.1654	-1.1654
	50	-25.9865	8.5467	-14.8753	-2.5421	-2.5421
c_s	-50	0.0054	0.0006	-0.0052	0	0
	-20	0.0078	0.0004	-0.0015	0	0
	20	-0.0087	-0.0055	0.0013	0	0
	50	-0.0035	-0.0034	0.0035	0	0

Observations from table 1.1:

1. $P(t_1,T)$ is slightly sensitive to changes in the values of parameters A, α, β, σ, h_1, h_2, c_s and it is moderately sensitive to changes in C and highly sensitive to changes in λ.
2. p is slightly sensitive to changes in the values of parameters A, α, β, σ, h_1, h_2, c_s and it is moderately sensitive to changes in C and highly sensitive to changes in λ.
3. From table 1, it is clearly seen that q is slightly sensitive to the changes in the values of parameters c_s, α, β and it is moderately sensitive to changes in A, σ, h_1, h_2, C and highly sensitive to changes in λ.
4. T and t_1 are insensitive to changes in the values in the values of the parameter c_s and slightly sensitive to changes in the values of parameters α, β and it is moderately sensitive to changes in A, σ, h_1, h_2, C and highly sensitive to changes in λ.

Table 1.2 Sensitivity Analysis with respect to the various parameters with shortages:

Parameter	% Change in Parameter	% Change in profit	% Change in p	% Change in Q	% Change in T
A	-50	5.8765	-0.0874	-7.8643	-7.8353
	-20	2.7643	-0.0065	-3.8653	-3.0953
	20	-2.8764	0.0175	2.7653	2.7542
	50	-4.8753	0.4325	6.9873	6.8873
Λ	-50	-94.8764	-46.9887	-55.3589	20.7654
	-20	-47.9764	-16.8775	-21.5436	5.8325
	20	64.8765	15.7765	21.9877	-2.8654
	50	157.0876	46.9864	53.9754	-6.7542
Σ	-50	0.0865	-0.0165	0.0954	0.1657
	-20	0.0245	-0.0086	0.0863	0.0854
	20	-0.0216	0.0056	-0.0753	-0.0674
	50	-0.0832	0.0176	-0.0876	-0.1543
h_1	-50	0.3567	-0.0976	0.6547	0.5467
	-20	0.1365	-0.0387	0.2435	0.2213
	20	-0.1299	0.0654	-0.3147	-0.3064
	50	-0.5467	0.0945	-0.6347	-0.5978
h_2	-50	0.7654	-0.7357	0.8975	0.8342
	-20	0.4326	-0.2754	0.6457	0.6107
	20	-0.2467	0.1879	-0.1235	-0.0543
	50	-0.6944	0.5634	-0.4265	-0.1423
A	-50	0.0543	-0.0087	0.0986	0.0843
	-20	0.0124	-0.0034	0.0057	0.0768
	20	-0.0223	0.0045	-0.0926	-0.0876
	50	-0.0416	0.0094	0.0988	-0.0546
B	-50	0.9870	-0.0198	0.0987	0.0765
	-20	0.7646	-0.0079	0.0087	0.0013
	20	-0.0456	0.0034	-0.0543	-0.675
	50	-0.0987	0.0094	-0.0954	-0.997
C	-50	30.7654	-8.9876	26.0778	7.9876
	-20	11.8765	-7.4323	8.9857	3.7536
	20	-10.0985	6.5855	-6.7899	-1.8589
	50	-26.9765	5.8868	-15.9765	-2.2467

Observations from table 1.2:

1. $P(t_1,T)$ is slightly sensitive to changes in the values of parameters α, β, σ, h_1, h_2 and it is moderately sensitive to changes in C and A and highly sensitive to changes in λ.
2. p is slightly sensitive to changes in the values of parameters A, α, β, σ, h_1, h_2 and it is moderately sensitive to changes in C and highly sensitive to changes in λ.
3. From table 2, it is clearly seen that q is slightly sensitive to the changes in the values of parameters α, β, σ, h_1, h_2 and it is moderately sensitive to changes in A, C and highly sensitive to changes in λ.
4. T is slightly sensitive to changes in the values of parameters α, β, σ, h_1, h_2 and it is moderately sensitive to changes in A, C and λ.

VI. CONCLUSION

A single item inventory model with constant replenishment rate, exponential demand rate, infinite time horizon, with exponential partial backordered rate, linearly increasing holding cost in both the warehouses and with the objective of maximizing the present worth of the total system profit, was developed in this paper. The total profit function is developed using five general costs: order cost, purchasing cost, holding cost, opportunity cost and shortage cost with sales Revenue. Order cost is fixed per replenishment and holding cost is linearly increasing with time. The cost of a backorder includes a fixed cost and a cost which is proportional to the length of time the backorder exists. Thus, we consider the practical situation where a salesperson offers compensation so as to not lose the sale and therefore establish an appropriate model for a retailer to determine its replenishment number and schedule when the backlogging rate is $e^{-\sigma x}$, where x is the waiting time and backlogging parameter σ is a positive constant. The proposed model allows not only the partial backlogging rate but also a constant deterioration rate. A comparative study between the results of the with-shortage case and without-shortage case is also done. In the numerical examples, it is found that the optimum average profit in with-shortage case is more than that of the without-shortage case. Hence the model with-shortage is considered to be better economically.

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