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New ZCW Complete Complementary Code Set and its Analysis

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***Abstract*— A new complete complementary code set with zero correlation window (ZCW) is constructed and it can be seen as a natural extension of conventional complete complementary code without ZCW. The construction method of this code set is motivated by that of Loose Synchronous (LS) code used in LAS-CDMA system. The main property of the new complementary code set of order 4 is that it can provide twice the number of code as the conventional LS code under the condition of same ZCW. The construction method of the new code set and the proof of the properties are shown in this paper.**

***Index Terms*— Zero Correlation Window (ZCW), Complete Complementary Code Set, Loose Synchronous (LS) code.**

I. INTRODUCTION

It is well known that the capacity of a CDMA system is tightly related to its access code design. If the aperiodic cross-correlation function (CCF) of an access code set is zero for any relative time shift and its aperiodic auto-correlation function (ACF) is zero for any relative time shift except for the origin, such access code set is ideal or perfect for a CDMA system. Consequently this CDMA system can reach the highest system capacity [1]. However, Welch [3] has proved that such ideal access code set does not exist. But complementary series [4] presented by Golay break up the constriction of Welch Bound, because each Golay code consists of two sequences, then two Golay codes with ideal ACF and CCF can be constructed as the result of cancellation between the two sequences. Later, Dr. Schweitzer extended Golay code's structure and presented generalized Complementary Code Set which can also be called complete complementary code of order N [9]. The structure of complete complementary code of order N is that each code in the set consists of $N = 2^m$ sequences ($m = 1$ is just the Golay complementary code) and there exist such $N = 2^m$ access codes with ideal ACF and CCF. Then D B Li constructed a complementary code set with zero correlation window [5] called Loose Synchronous (LS) code, i.e. its CCF is zero within a certain relative time shift and its ACF is also zero within this relative time shift except on the origin. This complementary code set can be seen as a natural extension of Golay complementary code, and it increases the number of available complementary codes due to the relaxation of perfect ACF and CCF constraint. LS code set is utilized in the LAS-CDMA (Large Area Synchronous CDMA) system.

In this paper, a new complete complementary code of order 4 with ZCW is constructed and the construction method comes from that of LS code [6]-[8]. The new code set can increase the number of conventional complete complementary code of order N, therefore it can promote CDMA system capacity. Theoretical proof shows that the new code set can double the number of complementary code of order 4 under the condition of the same ZCW compared with LS code. In addition, in the application of location technique [10]~[12], the new code set may improve the positioning accuracy, because of the ACF and CCF with ZCW. And in many other areas, this new ZCW complete complementary code set also has application, such as target detection, and parameter estimation in wireless sensor networks. Target detection and parameter estimation in wireless sensor networks have already attracted significant attentions as demonstrated in [9]-[36]. Some coding and decoding methods have been used in target detection and parameter estimation, such as in [17][18][27][28][29]. The new ZCW code set, if used in target detection and parameter estimation, may improve target positioning accuracy and detector performance.

This paper is organized as follows: Section II outlines the definition of complete complementary code set and gives two examples. In section III, the construction method of the new code set is demonstrated. Simulation results are listed in section IV. Section V concludes the paper.

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II. DEFINITION OF COMPLETE COMPLEMENTARY CODE SET AND INTRODUCTION TO LOOSE SYNCHRONOUS (LS) CODE

A complete complementary code set $\{U^1, U^2, \cdots, U^K\}$ consists of K codes and each code is composed by N sequences with each sequence's length equal to M. N is the order of this code set. In this paper, $M = 2^n$ where n is an arbitrary positive integer.

$\{U^1, U^2, \cdots, U^K\}$ can be expressed as

$$\begin{cases} U^1\left(U_1^1, & U_2^1, & \cdots\cdots, & U_N^1\right) \\ U^2\left(U_1^2, & U_2^2, & \cdots\cdots, & U_N^2\right) \\ & \vdots & & \\ U^K\left(U_1^K, & U_2^K, & \cdots\cdots, & U_N^K\right) \end{cases} \quad (1)$$

where $U_j^i = \left[u_{j,1}^i, u_{j,2}^i, \cdots, u_{j,M}^i\right]$ is the jth sequence of the ith code of $\{U^1, U^2, \cdots, U^K\}$, $u_{j,m}^i$ denotes the mth element of sequence U_j^i, i=1,2…K, j=1,2…N, m=1,2…M.

The aperiodic cross-correlation function (CCF) between U^i and U^k is defined as:

$$R_{U^i,U^k}(\tau) = \frac{1}{NM}\sum_{j=1}^{N} R_{U_j^i,U_j^k}(\tau) \quad (2)$$

where $R_{U_j^i,U_j^k}(\tau)$ is the CCF between U_j^i and U_j^k, which is defined as:

$$R_{U_j^i,U_j^k}(\tau)=\begin{cases}\sum_{m=\tau+1}^{M} u_{j,m}^{i} u_{j,m-\tau}^{*k} & 0\le\tau\le M-1\\ \sum_{m=1}^{M+\tau} u_{j,m}^{i} u_{j,m-\tau}^{*k} & -M+1\le\tau\le 0\\ 0 & |\tau|\ge M\end{cases} \quad (3)$$

where $u_{j,m-\tau}^{*k}$ is the complex conjugate of $u_{j,m-\tau}^{k}$. The aperiodic auto-correlation function (ACF) is substituting U_j^i for U_j^k. τ is the relative time shift.

Definition I: A code set in form of (1) with $K=N$ is called "a complete complementary code set of order N". If ACF of arbitrary code and CCF between arbitrary two codes satisfy the following equations:

$$R_{U^i}(\tau)=0,\quad \forall\tau\neq 0,\ \forall i=1,2,\cdots,N \quad (4)$$

$$R_{U_j^i,U_j^k}(\tau)=0,\ \forall i\neq k\in\{1,2,\cdots,N\} \quad (5)$$

Here, the order N denotes the number of sequences in one access code.

Definition II: If $\exists\tau_{\mathrm{w}}=\arg\min_{|\tau|}\left[R_{U_j^i,U_j^k}(\tau)\neq 0\right]$ and $R_{U_j^i}(\tau)=R_{U_j^k}(\tau)=0$, $\forall\tau\neq 0$, $\tau\le|\tau_{\mathrm{w}}|$, U^i and U^k is called a pair of ZCW complementary codes of order N with τ_{w}. τ_{w} is an one-side zero correlation window (ZCW).

A example of complete complementary code of order 4 with each sequence length 4 is given in figure2-1:

$$\{U_1^1,U_2^1,U_3^1,U_4^1\}=\{++++,+-+-,--++,-++-\}$$

$$\{U_1^2,U_2^2,U_3^2,U_4^2\}=\{-+-+,----,+--+,++--\}$$

$$\{U_1^3,U_2^3,U_3^3,U_4^3\}=\{--++,-++-,++++,+-+-\}$$

$$\{U_1^4,U_2^4,U_3^4,U_4^4\}=\{+--+,++--,-+-+,----,\}$$

Fig. 1 Example of complete complementary code of order 4

Loose Synchronous (LS) code[1] which was invented by D.B. Li can be seen as an extension of Golay code. Because of the existence of ZCW, the number of LS code can be much more than that of Golay code. LS code consists of two sequences called C sequence and S sequence the seed of C sequence and S sequence which is used for the generation of longer LS code is shown below:

$$\{C_1,S_1\}=\{++,+-\}$$

$$\{C_2,S_2\}=\{-+,--\}$$

The generation of LS code is given in Fig. 2:

$$\begin{matrix}C_1 & S_1\\ C_2 & S_2\end{matrix}\begin{cases}C_1C_2 & S_1S_2\\ C_1\overline{C_2} & S_1\overline{S_2}\\ C_2C_1 & S_2S_1\\ C_2\overline{C_1} & S_2\overline{S_1}\end{cases}$$

Fig. 2 The generation tree of LS code

Note that the operation of S sequence is the same as that of C sequence.

III. CONSTRUCTION METHOD OF A ZCW COMPLEMENTARY CODE SET OF ORDER 4

A. Notation Instruction

For the sake of simplicity, we use A,B,C,D to denote the corresponding sequence of $\{U^1,U^2,U^3,U^4\}$, that is to say, if A denote U_1^1, then B,C,D denote U_1^2,U_1^3,U_1^4 respectively, and if A denote U_2^1, then B,C,D denote U_2^2,U_2^3,U_2^4 respectively, and so on.

B. Construction Method

Inspired by the construction method of LS code, the construction method of complete complementary code of order 4 with ZCW is shown in Fig. 3. A,B,C,D used in Fig3 comes from the example in Fig. 1 as the initial seeds in the generation tree.

$$\begin{matrix}A\\B\\C\\D\end{matrix}\begin{cases}AB\\ A\overline{B}\\ BA\\ B\overline{A}\\ CD\\ C\overline{D}\\ DC\\ D\overline{C}\end{cases}\begin{cases}ABA\overline{B}\\ AB\overline{A}B\\ A\overline{B}AB\\ A\overline{B}\,\overline{A}\,\overline{B}\\ BAB\overline{A}\\ BA\overline{B}A\\ B\overline{A}BA\\ B\overline{A}\,\overline{B}\,\overline{A}\\ CDC\overline{D}\\ CD\overline{C}D\\ C\overline{D}CD\\ C\overline{D}\,\overline{C}\,\overline{D}\\ DCD\overline{C}\\ DC\overline{D}C\\ D\overline{C}DC\\ D\overline{C}\,\overline{D}\,\overline{C}\end{cases}$$

Fig. 3 The generation tree of complete complementary code of order 4 with ZCW.

Through the recursive structure of the generation tree, the number of complete complementary code of order 4 with ZCW is doubled in every stage. In the first column of the generation tree, there are four complete complementary codes of order 4. After some concatenation operation, eight complete complementary codes of order 4 with ZCW equal to four are constructed in the second column and the length of the code also doubled, i.e. each sequence length in the first column is equal to 4 and the sequence length in the second column is equal to 8. And then similar operation is utilized to further double the number of code and its length, but the length of ZCW remains the same as before. All of these properties is proved in section 3.2

C. Main Properties

Theory I : If A and B have perfect ACF and CCF, then AB and $A\overline{B}$ also have perfect ACF and CCF.

Proof:First,

$R_{AB,A\overline{B}}(\tau)=R_A(\tau)+R_{A,B}(\tau-4)+R_{B,\overline{B}}(\tau)+R_{A,\overline{B}}(\tau+4)$ Because A and B have perfect CCF, $R_{A,B}(\tau-4)=0$ and $R_{A,\overline{B}}(\tau+4)=0$.

Then $R_{AB,A\overline{B}}(\tau)=R_A(\tau)-R_B(\tau)$. Because A and B have perfect ACF,

$R_{AB,A\overline{B}}(\tau)=0$

When it comes to the ACF of AB and $A\bar{B}$, substituting $A\bar{B}$ to AB and vice versa. **Q.E.D**

Theory II: If A and B have perfect ACF and CCF, then AB and BA has a ZCW that is equal to the length of A or B, that is four. In addition, AB and $B\bar{A}$ also satisfy this property.

Proof: When $|\tau| < 4$, the proof is similar to that of theory I. But when $|\tau| = 4$, it is easy to see that A in AB will encounter A in BA and B in AB will encounter B in BA. Therefore, an ZCW that is equal to 4 is formed and the number of code can be doubled through this kind of concatenation. **Q.E.D.**

Theory III: If A, B, C, D have perfect ACF and CCF, then any concatenation of A and B and any concatenation of C and D has perfect ACF and CCF.

Proof: The proof is similar to that of theory I and theory II. **Q.E.D.**

IV. CONCLUSION

A construction method of ZCW complete complementary code set of order 4 has been proposed. This kind of code set is a natural extension of conventional complete complementary code set. It, of course, doubles the number of codes compared with the conventional LS code under the condition of the same one-side ZCW.

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