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# Distributed Estimation and Detection in Wireless Sensor Networks

**Zhenxing Luo**

*Abstract*— **Distributed estimation and detection are the two most important tasks of wireless sensor networks (WSNs). In the detection task, the fusion center needs to make a decision about the presence of a target. Usually, to make this decision, the fusion center uses a threshold. If the received signal is greater than the threshold, the fusion center considers the target is present. If the received signal is less than the threshold, the fusion center considers the target is absent. In the estimation problem, the fusion center will use a maximum likelihood estimation (MLE) method to estimate target location. In this MLE method, a threshold is needed for sensors to quantize information before sending information to the fusion center. This paper will investigate whether the two thresholds are identical. This problem is practically important because if the two thresholds are identical, the design of WSNs can be simplified.**

***Index Terms*—Distributed detection, distributed estimation, K-L distance, wireless sensor networks.**

## I. INTRODUCTION

Tasks and applications of wireless sensor networks (WSNs) have become popular research topics [1]-[13]. Among the tasks WSNs can perform are tracking, detection and estimation [14]. Target detection and target estimation are particularly important because they are widely used.

In a target detection problem, the fusion center, after collecting information from sensors, makes a decision about the presence of a target. Usually, the fusion center makes the decision according to a threshold. If the signal received from the sensor is greater than the threshold, the fusion center considers the target is present. If the signal received is less than the threshold, the fusion center considers the target is absent. This threshold is called detection threshold, and is an important parameter in the detection problem.

In an estimation problem, the fusion center, after collecting information from sensors, estimates the target position using a maximum likelihood estimation (MLE) method. To save energy, before sending information to the fusion center, sensors will quantize information according to a threshold. This threshold is called quantization threshold.

In this paper, we will investigate whether the quantization threshold and detection threshold are the same for various situations. This problem is practically important because one can simplify the design of WSNs if the two thresholds are identical. The main contribution of this paper is the comparison of detection threshold and quantization threshold.

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**Zhenxing Luo**, 721 Eastgate Ave, Apt. 2S, St. Louis.

This paper is organized in the following way. Section II presents distributed estimation in WSNs, followed by detection problem in WSNs in Section III. Section IV presents simulation setup and Section V provides results and discussion. Finally, Section VI delivers concluding remarks.

## II. DISTRIBUTED DETECTION IN WSNS

The system diagram of a WSN is shown in Figure 1.

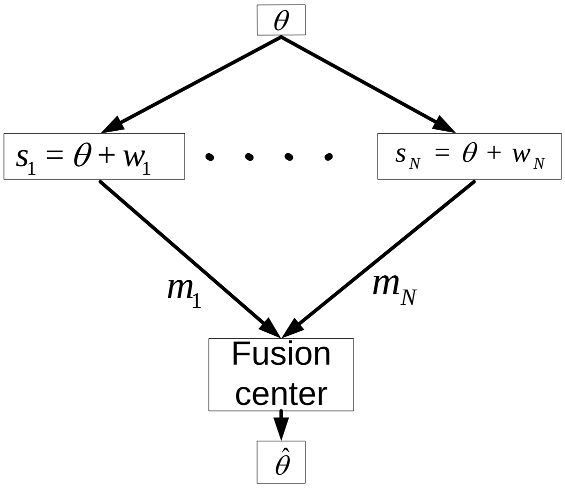


Figure 1. System diagram of a WSN

We use the same setup and formulation as in [15]-[17]. For completeness of the theory, the formulation presented in [15]-[17] is reproduced here. In the distributed estimation problem, an unknown parameter, $\theta$, is estimated based on information from a total number of $N$ sensors. Sensors are identical and $\theta$ follows the distribution $f(\theta)$, which is defined as:

$$f(\theta) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}. \tag{1}$$

The signal received at the $i$th sensor is $y_i$, which can be expressed as

$$y_i = \theta + w_i \tag{2}$$

where $\theta$ is the parameter to be estimated and $w_i$ is a Gaussian noise following the distribution $w_i \sim N(0,1)$. According to the quantization threshold $\eta_1$, sensors quantizes $y_i$ into a decision $D_i$ and the quantization process can be expressed as

$$D_i = \begin{cases} 0 & -\infty < s_i < \eta_1 \\ 1 & \eta_1 < s_i < -\infty \end{cases}. \tag{3}$$

The probability that $D_i$ takes value $l$ is

$$p_{il}(D_i = l|\theta) = \begin{cases} 1 - Q(\frac{\eta_1 - \theta}{\sigma}) & (l=0) \\ Q(\frac{\eta_1 - \theta}{\sigma}) & (l=1) \end{cases} \tag{4}$$

where $Q(x)$ is defined as

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt \tag{5}$$

After receiving decision vector

$$\mathbf{D} = [D_1, D_2, \cdots, D_{N-1}\ D_N], \tag{6}$$

the fusion center maximizes (7) to find a $\theta$, which can be expressed as

$$\ln p(\mathbf{D}|\theta) = \sum_{i=1}^{N} \sum_{l=0}^{1} \delta(D_i - l) \ln[p_{il}(\eta_1, \theta)] \tag{7}$$

$$\hat{\theta} = \max_{\theta} \ln p(\mathbf{D}|\theta). \tag{8}$$

For an unbiased estimator $\hat{\theta}$, the Posterior Cramer-Rao lower bound (PCRLB) is given by

$$E\{[\hat{\theta}(\mathbf{D}) - \theta][\hat{\theta}(\mathbf{D}) - \theta]^T\} \geq \mathbf{J}^{-1} \tag{9}$$

$$\mathbf{J} = -E\left[\nabla_\theta \nabla_\theta^T \ln p(\mathbf{D}, \theta)\right]. \tag{10}$$

The PCRLB for this distributed estimation problem can be found in [18][19]. For references, the PCRLB calculation is reproduced here. The PCRLB can be divided into two parts, $\mathbf{J}_d$ and $\mathbf{J}_p$, which can be expressed as

$$\begin{aligned} \mathbf{J} &= E\left[-\nabla_\theta \nabla_\theta^T \ln p(\mathbf{D}|\theta)\right] + E\left[-\nabla_\theta \nabla_\theta^T \ln p_0(\theta)\right] \\ &= \mathbf{J}_d + \mathbf{J}_p. \end{aligned} \tag{11}$$

The first part $\mathbf{J}_d$ can be calculated by

$$\begin{aligned} \mathbf{J}_d &= E\left[-\nabla_\theta \nabla_\theta^T \ln p(\mathbf{D}|\theta)\right] \\ &= \int_\theta \mathbf{J}_s(\theta) f(\theta) d\theta \end{aligned}. \tag{12}$$

In (12), $\mathbf{J}_s(\theta)$ can be calculated by

$$\mathbf{J}_s(\theta) = E\left[\frac{\partial^2 p(\mathbf{D}|\theta)}{\partial \theta^2}\right] = \sum_i \sum_l -\frac{1}{p_{il}} \left[\frac{\partial p_{il}}{\partial \theta}\right]^2 . \tag{13}$$

In (13), $p_{i0}(\eta_1, \theta)$ and $p_{i1}(\eta_1, \theta)$ are

$$p_{i0}(\vec{\eta_i}, \theta) = \int_{-\infty}^{\eta - \theta} \frac{1}{\sqrt{2\pi}} e^{-\frac{w^2}{2}} dw \tag{14}$$

$$p_{i1}(\vec{\eta_i}, \theta) = \int_{\eta - \theta}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{w^2}{2}} dw \tag{15}$$

The derivative of $p_{i0}(\eta_1, \theta)$ and $p_{i1}(\eta_1, \theta)$ with respect to $\theta$ can be expressed as

$$\Delta_0 = \frac{\partial p_{i0}(\eta_1, \theta)}{\partial \theta} = \frac{1}{\sqrt{2\pi}} e^{-\frac{(\eta_1 - \theta)^2}{2}} \times (-1) \tag{16}$$

$$\Delta_1 = \frac{\partial p_{i1}(\eta_1, \theta)}{\partial \theta} = \frac{1}{\sqrt{2\pi}} e^{-\frac{(\eta_1 - \theta)^2}{2}} . \tag{17}$$

Now, we have all elements of (13). The PCRLB is a useful performance criterion. The optimum quantization threshold can be determined by minimizing the PCRLB. Moreover, because PCRLB is the inverse of fisher information, minimizing PCRLB is equal to maximizing fisher information.

## III. DISTRIBUTED DETECTION IN WSNS

Target detection is also a classic problem in WSNs. The fusion center makes a decision about the presence of a target based on information from sensors. In this paper, because all sensors are identical, we only use one sensor. The detection problem is to decide either (18) is valid or (19) is valid

$$H_1 : y_i = \theta + w_i \tag{18}$$

$$H_0 : y_i = w_i . \tag{19}$$

For the detection problem, we still use the setup in (1)-(2).The detection threshold is $\eta_2$. If the received signal is greater than $\eta_2$, the fusion center considers the target is present ($u_1$). If the received signal is less than $\eta_2$, the fusion center considers the target is absent ($u_0$). The performance of a detection method can be measured by Kullback–Leibler (K-L) distance [20][21]. The K-L distance is defined as

$$\begin{aligned} &D\left(p(u_i|H_1) \| p(u_i|H_0)\right) \\ &= \sum_{i \in \{0,1\}} p(u_i|H_1) \log \frac{p(u_i|H_1)}{p(u_i|H_0)} \end{aligned} . \tag{20}$$

Elements of (20) can be calculated by

$$\begin{aligned} &p(u_1|H_1) \\ &= p(\theta + w_i > \eta_2) \\ &= p(w_i > \eta_2 - \theta) \\ &= \int_{-\infty}^{+\infty} \int_{\eta_2 - \theta}^{+\infty} \frac{1}{\sqrt{2\pi}} \exp(-\frac{w_i^2}{2}) f(\theta) dw_i d\theta \end{aligned} \tag{21}$$

$$\begin{aligned} &p(u_0|H_1) \\ &= p(\theta + w_i < \eta_2) \\ &= p(w_i < \eta_2 - \theta) \\ &= \int_{-\infty}^{+\infty} \int_{-\infty}^{\eta_2 - \theta} \frac{1}{\sqrt{2\pi}} \exp(-\frac{w_i^2}{2}) f(\theta) dw_i d\theta \end{aligned} \tag{22}$$

$$\begin{aligned} &p(u_1|H_0) \\ &= p(w_i > \eta_2) \\ &= \int_{-\infty}^{+\infty} \int_{\eta_2}^{+\infty} \frac{1}{\sqrt{2\pi}} \exp(-\frac{w_i^2}{2}) f(\theta) dw_i d\theta \end{aligned} \tag{23}$$

$$\begin{aligned} &p(u_0|H_0) \\ &= p(w_i < \eta_2) \\ &= \int_{-\infty}^{+\infty} \int_{-\infty}^{\eta_2} \frac{1}{\sqrt{2\pi}} \exp(-\frac{w_i^2}{2}) f(\theta) dw_i d\theta \end{aligned} \tag{24}$$

We can insert (21) to (24) into (20) to derive the K-L distance.

## IV. SIMULATION SETUP

We need to find an optimum threshold $\eta_1$ to maximize the fisher information (minimize PCRLB) for the distributed estimation problem. For detection problem, we need to find an optimum threshold $\eta_2$ to maximize the K-L distance. To see whether $\eta_1$ is equal to $\eta_2$, we need to find the optimum $\eta_1$ and $\eta_2$ for two different types of distributions: $\theta$ follows uniform distribution $\theta \in u[-1\ \ 1]$ and $\theta$ follows exponential distribution ($\lambda = 1$).

## V. RESULTS AND ANALYSIS

For uniform distribution, results are shown in Figure 2 and Figure 3. We can see that the fisher information achieved maximum value around 0.6 while the K-L distance achieved maximum value at 0. For exponential distribution, results are shown in Figure 4 and Figure 5. We can see that the fisher information achieved maximum value around 0.6 and the K-L distance achieves maximum point around 1.9. Therefore, we can conclude that the detection threshold and quantization threshold are not the same for $\theta$ following uniform distribution or exponential distribution.

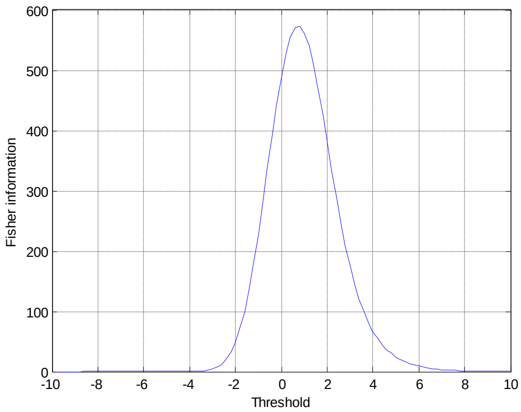


Figure 2. Fisher information for uniform distribution

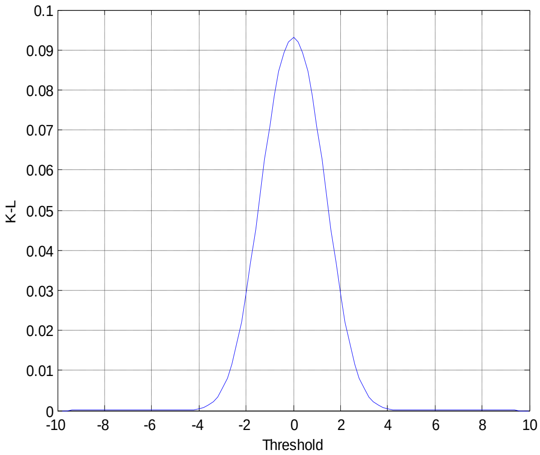


Figure 3. K-L distance for uniform distribution

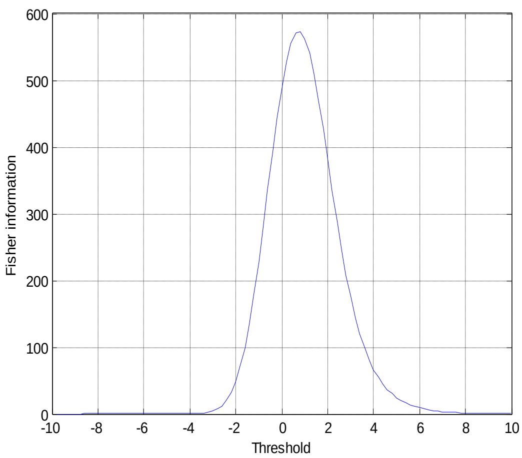


Figure 4. Fisher information for exponential distribution ($\lambda = 1$)

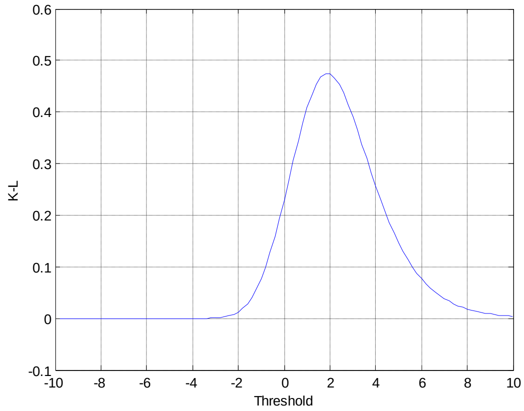


Figure 5. K-L distance for exponential distribution ($\lambda = 1$)

## VI. CONCLUSION

In this paper, we found the detection threshold and quantization threshold for $\theta$ following uniform distribution or exponential distribution. Results showed that the quantization threshold and detection threshold are not the same.

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