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Conjugate Gradient Based Mmse Filter for Cfo Compensation In Uplink Orthogonal Frequency Division Multiple Access Systems

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Abstract: Carrier Frequency Offset (CFO) compensation is very important for reliable detection of transmitted data in uplink Orthogonal Frequency Division Multiple Access (OFDMA) systems. In this paper we proposed a low-complexity CFO compensation algorithm based on the Minimum Mean Square Error (MMSE) criterion for uplink OFDMA systems. The proposed algorithm employs a Conjugate Gradient (CG) method which iteratively finds the MMSE solution. In this paper we are presenting the proposed method by comparing with the existing direct MMSE method and we show that CFO can be compensated with substantially reduced computational complexity by applying the CG method.

Index terms- Carrier Frequency Offset (CFO), Orthogonal Frequency Division Multiple Access (OFDMA), Conjugate Gradient (CG).

I. INTRODUCTION

Wireless communication systems have become an integral part of our lives. Orthogonal Frequency Division Multiplexing (OFDM) has shown to be a successful technique in combating the frequency-selective and multipath channel in digital communication. OFDM in its primary form is considered as a digital modulation technique and not a multi-user channel access technique. Multiple access is a challenging issue in designing OFDM systems to fulfil the requirement of future communications OFDM can be combined with existing multiple access techniques, such as Time-Division Multiple Access (TDMA), Frequency-Division Multiple Access (FDMA), and Code-Division Multiple Access (CDMA). Apart from these methods, the intrinsic orthogonal carrier nature of OFDM provides a unique multiple access capability, in which multiple subcarriers are assigned to multiple users for simultaneous transmission. This is termed as Orthogonal FDMA (OFDMA).

II. OFDMA

2.1 Overview of OFDMA

In recent years, Orthogonal Frequency Division Multiple Access (OFDMA) has emerged as the physical layer for different wireless network standards. OFDMA employs multiple closely spaced subcarriers. The subcarriers are divided into groups of subcarriers. Each group is named a sub-channel. The subcarriers that form a sub-channel need not be adjacent. OFDMA provides multiplexing operation of data streams from multiple users onto the downlink sub-channels and uplink multiple access by means of uplink sub-channels. The multiple access is achieved by assigning different OFDM sub-channels to different users. In the downlink, a sub-channel may be intended for different receivers. In the uplink, a transmitter may be assigned for one or more sub-channels [1] [2].

OFDMA achieves high spectral efficiency in a multi-user environment as it divides the total available bandwidth into narrow orthogonal sub-bands. The divided sub-bands are allocated to Mobile Users (MUs) according to carrier assignment schemes such as Sub-band based Carrier Assignment Scheme (SCAS), an Interleaved CAS (ICAS) and a Generalized CAS (GCAS) [5].

OFDMA is extremely sensitive to timing errors and Carrier Frequency Offset (CFO) between the incoming waveform and the local references used for signal demodulation. CFO is caused by a Doppler shift or mismatch between the frequencies of the transmitter and the receiver oscillators. Inaccurate compensation of the frequency offset destroys orthogonality among subcarriers and produces Inter-Carrier Interference (ICI) as well as Multi Access Interference (MAI) which leads to unacceptable Bit Error Rate (BER) performance. Thus, CFO compensation is important for reliable detection of transmitted data [6].

2.2 CFO in uplink OFDMA system

With a multiuser scenario, in uplink, there are many mobile stations communicating with the Base Station, and available resources are shared by these mobile devices. Base Station receives the signals from all of these devices at the same time. In this paper, we consider an uplink OFDMA system with N subcarriers and K mobile users. The CFO of the K-th mobile user is defined as ε_k which is normalized by the subcarrier spacing. In addition, the set C_k represents the subcarrier indices of the $k-th$ MU where $C_k \cap C_1 = \varphi$. for $k \neq l$. To simplify this presentation, we suppose that the cardinality of $C_k = \frac{N}{K}$. We defined channel impulse vector h_k as $h_k = [h_{0,k},\ h_{1,k}, h_{2,k}, h_{3,k}, h_{L-1,\ k}]^{\ T}$ where $h_{i,k}$ has an independent and identically distributed complex Gaussian distribution and L stands for the channel length. We denote $F_l = f_0, f_1, f_2, \ldots. f_{N-1}$ as the N×N DFT matrix with $f_i = \frac{1}{\sqrt{N}}[1\ e^{\frac{-j2\pi i}{N}} \ldots\ldots e^{\frac{-j2\pi i(N-1}{N})}]^T$, and we define $F_l = f_0, f_1, f_2, \ldots. f_{N-1}$. Then, the diagonal channel matrix H_k is given as $H_k = \sqrt{N} diag\{F_L h_k\} = diag\ \{H_{0,k}, H_{1,k}, H_{2,k}, H_{N-1,k}\}$ where $H_{i,k}$ is the channel gain of the $i-th$ subcarrier for the $k-th$ MU. Moreover, the transmitted signal of the $k-th$ MU is obtained as $X_k = [X_{0,k},\ X_{1,k}, X_{2,k}, X_{3,k}, X_{L-1,\ k}]^{\ T}$ where $X_{m,k} = 0$ for $m \neq\ C_k$m and $E[|X_{m,k}{}^2|]\ =\ 1$ for$m\ \epsilon\ C_k$. The received signal vector $r\ = [r_0,\ r_1, r_2, r_3, r_{N-1}]^{\ T}$after the cyclic prefix removal is computed as

$$r = \sum_{k=1}^{K} \boldsymbol{\Gamma}(\epsilon_k) F^{\dagger} H_k X_k + W \;\ldots\ldots\ldots\ldots\ldots (1)$$

where $\boldsymbol{\Gamma}(\epsilon_k) = diag\left\{1, e^{\frac{j2\pi\epsilon_k}{N}}, \ldots\ldots e^{\frac{j2\pi(N-1)\epsilon_k}{N}}\right\}$ represents a diagonal matrix and $w = [w_0, w_1, w_2, w_3, w_{N-1}]^T$ indicates the complex additive white Gaussian noise (AWGN) vector with zero mean and covariance matrix $\sigma_w^2 I_N$. A diagonal matrix Ψ_k is defined as $[\Psi_k]_{i,i} = 1$ for $i \in c_k$ and $[\Psi_k]_{i,i} = 0$ otherwise. For brevity, the composite transmitted data for the MUs is denoted as $x = \sum_{k-1}^{K} \Psi_k x_k = [\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_{N-1},]$ where $\bar{X}_i = \bar{X}_{i,k}$ for $i \in c_k$. Similar to x, the composite channel frequency-response is given as $H = \sum_{k-1}^{K} \Psi_k H_k = [\bar{H}_1, \bar{H}_2, \ldots, \bar{H}_{N-1},]$ where $\bar{H}_i = \bar{H}_{i,k}$ for $i \in c_k$ Then, the DFT output of (1) is written as

$$\bar{r} = \left(\sum_{k=1}^{K} C(\epsilon_k)\psi_k\right) H_x + \bar{W} = Qu + \bar{W} \;\ldots\ldots\ldots\ldots\ldots (2)$$

Where $C(\epsilon_k) = F\,\Gamma(\epsilon_k)F^{\dagger}$ is a circulant matrix and $Q = \sum_{k-1}^{K} C(\epsilon_k)\Psi_k$ represents the interference matrix, and $U = Hx$ and $\bar{w} = Fw$ respectively. The interference matrix Q characterizes the normalized interference generated by multiple CFOs in the frequency domain, and the received signal vector $\bar{r}$ is contaminated by the interference among subcarriers [5].

III. CFO COMPENSATION ALGORITHMS

3.1 Direct MMSE method

The least squares or MMSE criterion can be applied to suppress the interference among sub-carriers on the basis of the estimated CFO of each mobile user at the uplink receiver. In MMSE, the second order statistics of the signals and the noise are known to the uplink receiver. Here, we assume that the Carrier Frequency Offsets (CFOs) of each mobile user are known at the base station and also we assume that the noise on each sub-carrier is additive white Gaussian with zero mean and covariance σ^2_n.

After the linear Minimum Mean Square Error (MMSE) filter is applied, the Carrier Frequency Offset (CFO) compensated signal is given by

$$\hat{u} = (Q^{\dagger}Q + \sigma_{\omega}^2 I_N)^{-1} Q^{\dagger}\bar{r} \;\ldots\ldots\ldots\ldots (3)$$

Where, $Q^{\dagger}$ represents complex conjugate transpose of interference matrix Q, $\sigma_{\omega}^2 I_N$ represent Covariance matrix and $\bar{r}$ indicate Received signal vector. Because the interference among subcarriers is minimized by the MMSE filter, we can detect transmitted data by using one-tap equalizer.

CFO compensation by the Minimum Mean Square Error (MMSE) filter is simple and efficient. However, the computation of the MMSE filter requires an inverse operation of an N x N interference matrix whose size equals the number of sub-carriers as seen in (3). Thus, its required memory storage and computational complexity increase dramatically as N increases [1] [8].

3.2 Proposed conjugate gradient (CG) method

The evaluation of the MMSE filter requires a matrix by matrix multiplication, an inverse operation and storage for two NXN matrices. To resolve the complexity and storage issues, the Conjugate Gradient (CG) method is employed, to obtain a solution $\hat{u}$ for equation (3), which is rewritten as

$$M\hat{u} = b \;\ldots\ldots\ldots\ldots\ldots (4)$$

Where, $M = Q^{\dagger}Q + \sigma_{\omega}^2 I_N$ and $b = Q^{\dagger}\bar{r}$.

M is an NXN Hermitian positive-definite matrix, and b and z denote the complex vectors of length N. Then, the detailed algorithm for solving the equation (3) is as follows: The input vector $u^{(0)}$ can be an approximate initial solution or zero.

$g^{(0)} = b - Mu^{(0)}$ and $i = 0$............ (5)

$d^{(0)} = g^{(0)} \;\ldots\ldots\ldots\ldots\ldots (6)$

While $i < i_{max}$ and $G(g^{(i)}) > \delta^2$

$$\alpha^{(i)} = \frac{g^{(i)\dagger} g^{(i)}}{d^{(i)\dagger} d^{(i)}} \;\ldots\ldots\ldots\ldots\ldots (7)$$

$$u^{(i+1)} = u^{(i)} + \alpha^{(i)} d^{(i)} \;\ldots\ldots\ldots\ldots\ldots (8)$$

$$g^{(i+1)} = g^{(i)} - \alpha^{(i)} M d^{(i)} \;\ldots\ldots\ldots\ldots (9)$$

$$\beta^{(i)} = \frac{g^{(i+1)\dagger} g^{(i+1)}}{g^{(i)\dagger} g^{(i)}} \;\ldots\ldots\ldots\ldots (10)$$

$$d^{(i+1)} = g^{(i+1)} + \beta^{(i+1)} d^{(i)} \;\ldots\ldots\ldots\ldots\ldots (11)$$

$i = i + 1$

Where the superscript i denotes the iteration number, i_{max} represents the maximum iteration number, δ stands for the tolerance of the solution, and $G(g^{(i)})$ is defined as $(g^{(i)\dagger} g^{(i)})/(g^{(0)\dagger} g^{(0)})$, $g^{(i)}$ and $d^{(i)}$ are referred to as the residual vector and the search direction, respectively.

Generally the last residue vector is obtained by minimizing the metric following the search direction. In practice, the condition for obtaining last residue vector is slightly modified. Instead of minimizing the metric following the search direction, make the next residue orthogonal to the previous residue. In this way, the residues form an orthogonal basis, and each residue is conjugated to all search directions before the previous step. Minimization of the metric along the search direction will not be obtained in each step, but convergence is still fast. With exact computation, the algorithm finds the exact solution in last search direction, and only one matrix vector multiplication [7].

This algorithm requires storage of only $g^{(i)}$, $g^{(i+1)}$ and $d^{(i)}$ ie.., the last two residual vectors and the last search direction and one matrix by vector multiplication at each iteration are required. Thus, the total storage requirement is reduced.

The total storage requirement decreases, while there is no evidence of complexity reduction, since the total complexity is basically determined by the number of iterations. If the distribution of the eigenvalues of the M is known, the maximum iteration number required for the convergence can be estimated. $\lambda_{min}(D)$ and $\lambda_{max}(D)$ are defined as the minimum and the maximum eigenvalues of D, respectively.

1. After performing enough iterations to satisfy $G(g^{(i)}) < \delta^2$, the maximum number of iterations required to achieve the given accuracy is

$$i_{max} \leq \frac{1}{2}\sqrt{k}\ln\left(\frac{2}{\delta}\right) \leq N \;\ldots\ldots\ldots\ldots\ldots (12)$$

Where $k \triangleq \lambda_{max}(M)/\lambda_{min}(M)$ is the spectral condition number of M. It can be noted that CG method can always find an exact solution within N iterations.

2. If the number of distinct eigenvalues is N_e, the number of iterations for an exact solution is at most N_e.

3. The CG method finds the solution more quickly when eigenvalues are clustered together [3] [5].

If the eigenvalues of M satisfy the above 2 or 3, the required iteration number is much smaller than the maximum iteration numbers evaluated in equation (12).

IV.EXPERIMENTAL RESULTS AND DISCUSSIONS

We analyzed the convergence in two different carrier assignment schemes ICAS and GCAS. For interleaved carrier assignment scheme (ICAS) the covariant matrix of the MMSE filter is Hermitian block-circulant and we prove that for ICAS, the maximum iteration number for computing an exact solution is at most the same as the number of users. Moreover, for generalized carrier assignment scheme (GCAS), we show that the CG method can find a solution with far fewer iterations than the number of subcarriers.

Fig.1 compares the computational complexity of the CG method and direct MMSE method according to the number of subcarriers. We considered the computational complexity required for the GCAS for a fair comparison. Here, we assumed K=8 and p=N/K where N_{iter} denotes the number of iterations. Fig.1 shows that the CG method exhibits the lowest complexity, whereas the direct MMSE method has steep slope with respect to N.

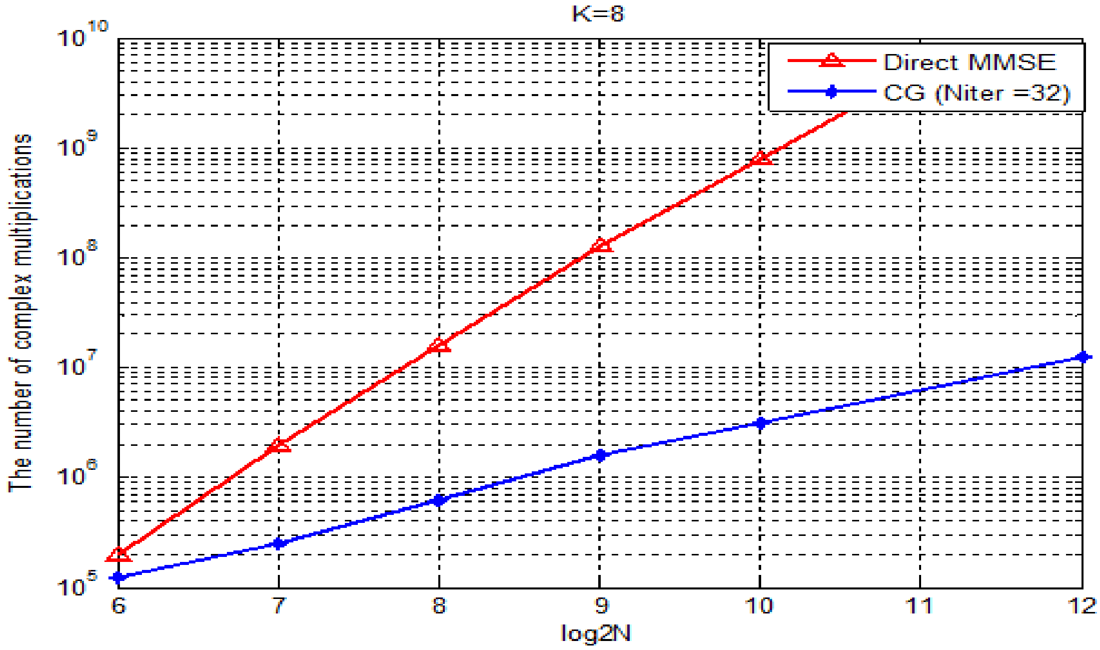

Fig.1. Computational loads of CFO compensation Algorithms CG and direct MMSE.

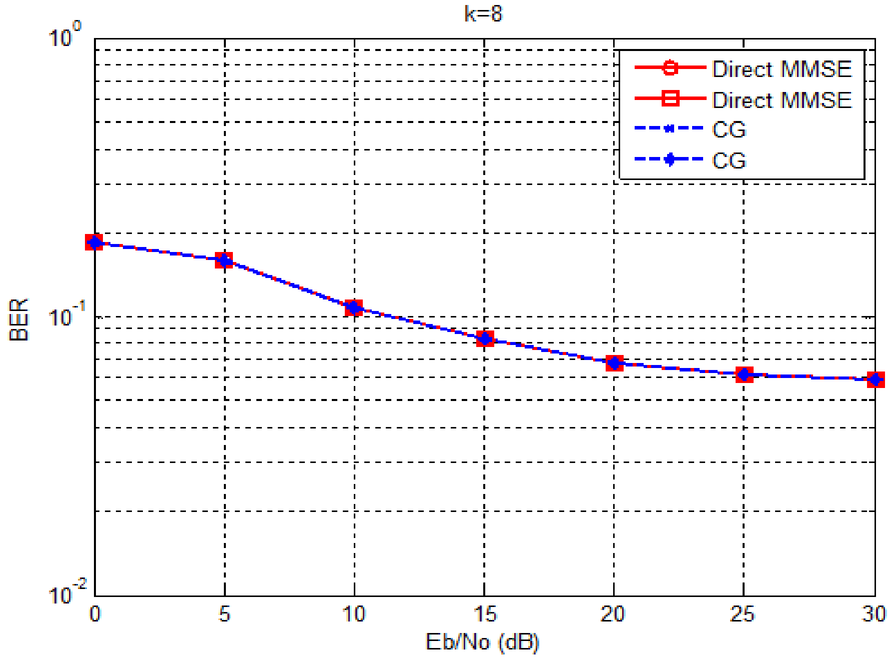

Fig.2. BER of Direct MMSE and CG for ICAS.

Fig.2 Compares the BER performance of the CG method with that of the Direct MMSE for the ICAS. We see that the BER performance of the CG method is identical to that of the Direct MMSE. Consequently, for the ICAS, *N*iter can be set to *K* without any performance loss. In this case, the computational complexity of the CG method is given by $O(K^2N \log N)$, whereas that of the Direct MMSE is $O(N(N+K^3))$. Therefore, the required complexity of the former is much lower than that of the latter.

Fig. 3 presents the average iteration number of the CG method for the ICAS. We see that the average iteration number for $K = 32$ is much smaller than 32, which indicates that a solution that satisfies $\delta \leq 10^{-4}$ is found within 32 iterations. Hence, the average computational complexity of the CG is much less than that presented in Fig. 2 which accounts for the worst complexity. Moreover, it can be seen that the average iteration number decreases at low SNR.

Fig. 4 illustrates the BER performance of the CG and the PCG methods for the GCAS. The CG method exhibits almost the same performance as the Direct MMSE in the low and moderate SNR regions with iterations much smaller than 512. However, for a large *K*, the CG method exhibits a slight BER performance loss at high SNR because of the limitation on *N*iter.

Fig. 5 presents the average iteration number of the CG method for the GCAS for different number of mobile users. the average iteration number is reduced as the SNR decreases, while the performance of the Direct MMSE is achieved. This can be explained by the fact that the residual error9 of the CG is masked by the noise in low and moderate SNR regions. Thus, we can further reduce the iteration numbers by adaptively adjusting the tolerance level.

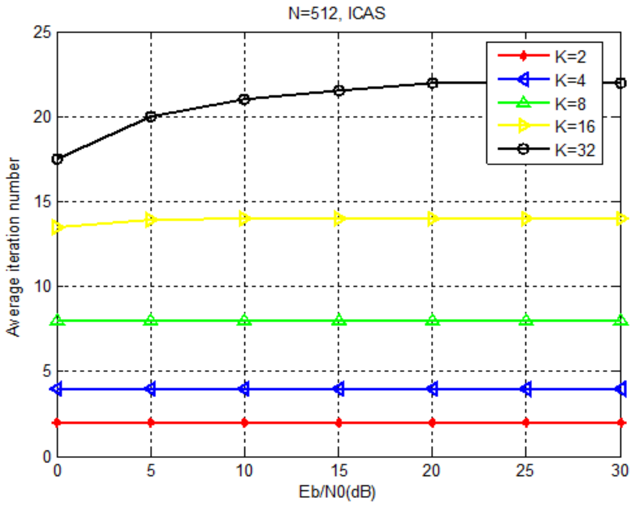

Fig.3. Average iteration number of CG for ICAS

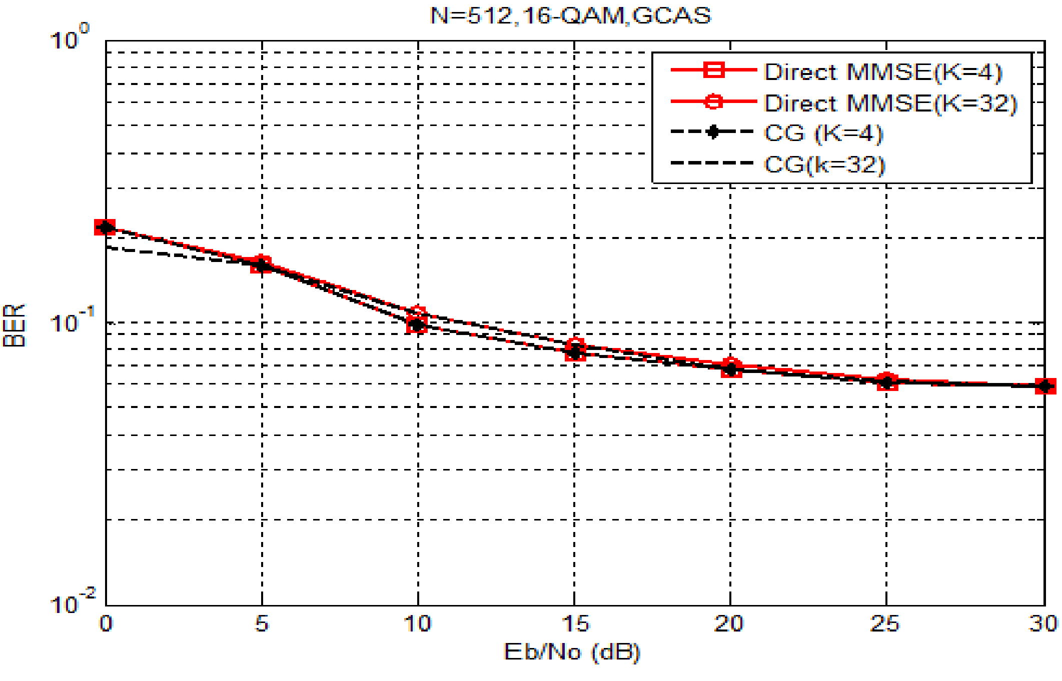

Fig.4. BER of Direct MMSE and CG for GCAS

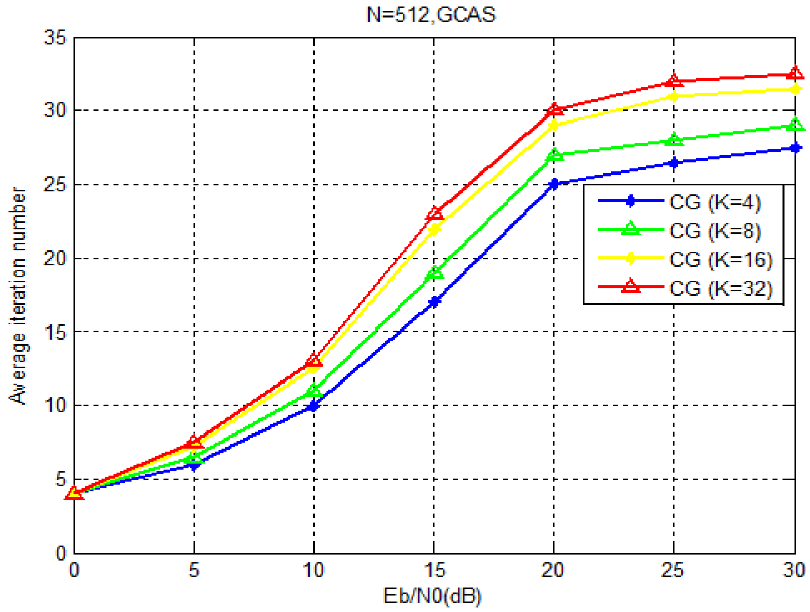

Fig.5. Average iteration number of CG for GCAS

V. CONCLUSION

Hence, a solution based on the Conjugate Gradient (CG) method that minimizes the computational complexity required to compensate the Carrier Frequency Offset (CFO) in uplink OFDMA systems is proposed. Our simulation results show that the CG method converges to a solution with far less iterations than the number of subcarriers. As a result, CFO compensation can be achieved with a significantly reduced computational complexity.

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