International Journal of Inventive Engineering and Sciences (IJIES)
ISSN: 2319-9598 (Online), Volume-12 Issue-8, August 2025

Numerical and Matrix-Based High-Performance Solver of Coupled Differential Equation Models under the Consideration of Neurocomputing

Shakhzod Tashmetov

Abstract**:** ***One interesting way to address the complexity of coupled differential equation models is to combine neurocomputing approaches with matrix-based high-performance solvers. In this work, we investigate how neurocomputing topologies and sophisticated numerical methods can be combined to overcome the computational challenges associated with using such models. Differential equation coupling often results in nonlinear dynamics with complex variable interactions, presenting substantial challenges for conventional computing methods. Our goal is to enhance the scalability, accuracy, and computational efficiency of complex system simulations by integrating the power of matrix-based solvers with the adaptive learning capabilities of neural networks. This multidisciplinary method has applications in physics, biology, engineering, economics, and other domains. It also promises to further our knowledge of complex dynamical systems. This paper aims to provide new pathways for approaches that can help decipher coupled differential equation models and lead to breakthroughs across a range of fields by combining theoretical analysis, algorithm development, and empirical validation to solve coupled differential equations using neurocomputing and/or matrix computation methods.***

Keywords**:** **Coupled** ***Differential Equations (CDEs), Dynamical Systems, Matrix Operations, Nonlinear Dynamics, Neurocomputing.***

Abbreviations:
CDEs: Coupled Differential Equations
ANNs: Artificial neural networks

I. INTRODUCTION

Coupled differential equations (CDEs) are mathematical models used to study the interactions and changes in multiple variables over time or in response to another independent variable. [1]. CDEs are typically revealed in systems that occur in biology, physics, engineering, economics, and other fields [2]. Applications of CDEs can be formidable to solve or process because the included variables are coupled and highly nonlinear, but CDEs are excellent tools for modelling some complex systems [3]. Numerical processing of coupled differential equations in practice is often available for large-scale systems of often-limited dimensions, and can require significant computing power.

Manuscript received on 31 July 2025 | First Revised Manuscript received on 04 August 2025 | Second Revised Manuscript received on 07 August 2025 | Manuscript Accepted on 15 August 2025 | Manuscript published on 30 August 2025.

*Correspondence Author(s)
Shakhzod Tashmetov*, Department of Television and Radio Broadcasting Systems, Tashkent University of Information Technologies named after Muhammad al-Khwarizmi, Tashkent, Uzbekistan. Email ID: shaxzod.tashmetov@gmail.com, ORCID ID: 0009-0009-3155-4771.

Most often, when considering applications of CDEs as applied in reality or concretely, with current 'derivative' codified approaches, it is feasible to consider old generation approaches, such as finite difference, finite element, spectral method, etc., being more computationally expensive and less scalable to very large-scale problems [4].

One possible way to overcome these limitations is to utilise neurocomputing with matrix-based high-performance solvers [5]. Artificial neural networks (ANNs) have demonstrated significant potential for neurocomputing, enabling the learning of complex patterns in data and adapting to their environment [6]. By this capability, neural networks are beneficial for approximating nonlinear dynamics and accelerating the solution process of coupled systems through the analysis of bifurcation diagrams and Lyapunov exponents [7]. In contrast, practical solutions to linear algebraic equations rely on matrix-based solvers, which form the basis of many traditional numerical methodologies. The combination of these two methods can leverage the adaptive learning capacity of neural networks and the computational efficiency of matrix solvers.

The fundamental idea underlying this multidisciplinary framework is to integrate neural network architectures and matrix operations to enhance the scalability, accuracy, and efficiency of coupled differential equation models. Specifically, matrix solvers will be responsible for the computationally intensive work required for larger systems. At the same time, neural networks can be trained to either approximate solutions or accelerate convergence to the desired solutions [8]. When deployed together, we aim to address critical issues associated with nonlinear differential equations, including computational challenges, error propagation, and lengthy simulation timelines.

The potential for applying this integrated method to many different areas is exciting. It can be used within engineering to model complex systems, such as fluid dynamics and structural behaviour, empirically [9]. Coupled models are frequently employed in biology to simulate population dynamics and biochemical processes [10]. Coupled models are also used in economics to predict economic outcomes and simulate market behaviour. The prospect of providing more thorough insights into system dynamics, with the ability to solve these models, will open new areas of research and analysis, and provide opportunities to create predictive models for optimisation and informed decision-making.

This study presents a framework for efficiently and

Published By:
Blue Eyes Intelligence Engineering
and Sciences Publication (BEIESP)

accurately solving digital twin coupled differential equation models, which are composed by combining neurocomputing techniques and matrix-based solvers. We demonstrate the advantages of this method for large, nonlinear systems through theoretical analysis, algorithm development, and experimental assessments, which may also help resolve problems associated with industrialisation.

This article is organised as follows: We cover basic concepts related to differential equations, as discussed in Section 2. The numerical approach for solving the system, along with the associated matrix-based solvers, is described in Section 3. The experimental findings, along with a comparison between the distinct approach and traditional algorithms, are presented in Section 4. We examine the results and provide some suggestions for further study in the conclusion, which is found in Section 5.

II. BASIC CONCEPTS RELATED TO DIFFERENTIAL EQUATIONS

The following coupled differential equations (DE) in (1) will be under consideration:

$$\begin{cases} \frac{d^2x_1}{dt^2} + a_1x_1 + a_2x_2 - a_4x_2^2 = a_3 \\ \frac{d^2x_2}{dt^2} + b_1x_2 + b_2x_1 - b_4x_1^2 = b_3 \end{cases} \quad \dots \quad (1)$$

The parameters of (1) are defined as follows: $a_1 = 1.5$; $a_2 = -0.1$; $a_3 = 10$; $a_4 = 0.0001$; $b_1 = 0.1$; $b_2 = -1$; $b_3 = -100$; $b_4 = 0.0001$; $0 \le t \le 500$.

The order of a differential equation is the order of the highest differential coefficient. In this case, the order of (1) is two second-order differential equations, which will be a fourth-order DE.

The degree of a differential equation is the power of the highest derivative. In our case, the degree of (1) is one in both differential equations. The variables can be defined as:

- Independent variable, which is "t" in (1);
- Dependent variables which are "x_1" and "x_2" in (1).

The most basic classification of differential equations is:

- Ordinary differential equations (ODEs) are equations where the derivatives are taken with respect to only one variable. That is, there is only one independent variable.
- Partial differential equations (PDEs) are equations that depend on partial derivatives of several variables. That is, there are several independent variables.

By definition, the ODE (1) will be described as a coupled ODE. The differential equations are as in (2):

$$\begin{cases} \dot{x}_1 = -x_1 - x_3 \\ \dot{x}_2 = 4x_1 - x_2 - 3x_3 \\ \dot{x}_3 = 2x_1 - 4x_3 \end{cases} \quad \dots \quad (2)$$

It is called coupled, whereas the system involves two or more variables. And, the differential equations as in (3):

$$\begin{cases} \dot{y}_1 = -3y_1 \\ \dot{y}_2 = -y_2 \\ \dot{y}_3 = -2y_3 \end{cases} \quad \dots \quad (3)$$

It is called uncoupled. The terminology uncoupled means that each differential equation in (3) depends on exactly one variable, e.g., $\dot{y}_1 = -3y_1$depends only on variable y1. In a coupled system, one of the equations must involve two or more variables.

Linear and nonlinear differential equations:

If the dependent variable and all of its derivatives appear linearly as in (4):

$$a_0(x)y + a_1(x)y' + a_2(x)y'' \cdots + a_n(x)y^{(n)} = b(x) \quad \dots \quad (4)$$

or if they are not multiplied together or squared, or if they are not part of transcendental functions such as sine, cosine, exponentials, etc., the differential equation is linear. Otherwise, the system is nonlinear. In terms of the concept of linearity, (1) is a nonlinear, time-invariant and autonomous system. An autonomous differential equation is an equation of the form as in (5):

$$\frac{dy}{dt} = f(y) \quad \dots \quad (4)$$

A time-variant system is a system whose input and output characteristics change over time. Time-Invariant System - If the input and output characteristics of a system do not change with time, and when all the coefficients are constant, the system is called a time-invariant system.

III. EXPERIMENTAL RESULTS OF SOLVING THE SYSTEM WITH DIFFERENT METHODS

A. The Numerical Approach of Solving the System

To solve the system, first, the dependent variables $x_1 = x$ and $x_2 = y$ It needs to be assigned, and it can be reformulated as in (6). Then, we have to apply Euler's method to equation (1). By using this method, (1) will be split into four equations of order 1, as in (7).

$$\begin{cases} \frac{d^2x}{dt^2} = -a_1x - a_2y + a_4y^2 + a_3 \\ \frac{d^2y}{dt^2} = -b_1y - b_2x + b_4x^2 + b_3 \end{cases} \quad \dots \quad (6)$$

By applying Euler, (7) will be written as follows:

$$\begin{cases} \frac{dx}{dt} = z \\ \frac{dz}{dt} = -a_1x - a_2y + a_4y^2 + a_3 \\ \frac{dy}{dt} = v \\ \frac{dv}{dt} = -b_1y - b_2x + b_4x^2 + b_3 \end{cases} \quad \dots \quad (7)$$

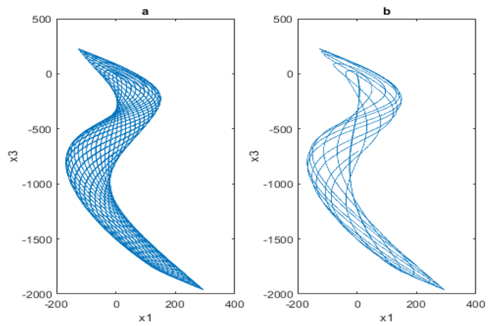
Next, the system variables need to be assigned to MATLAB Script, as can be seen from the following system as in (8):

$$\begin{cases} \frac{dx_1}{dt} = x_2 \\ \frac{dx_2}{dt} = -a_1x_1 - a_2x_3 + a_4x_3^2 + a_3 \\ \frac{dx_3}{dt} = x_4 \\ \frac{dx_4}{dt} = -b_1x_3 - b_2x_1 + b_4x_1^2 + b_3 \end{cases} \quad \dots \quad (8)$$

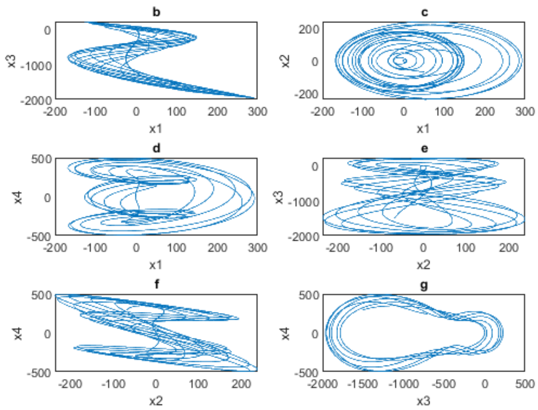
Plot of all solutions and phase portraits of (1) will be demonstrated in Fig. 1 – Fig. 1.2.

Retrieval Number: 100.1/ijies.H111712080825
DOI: 10.35940/ijies.H1117.12080825
Journal Website: www.ijies.org

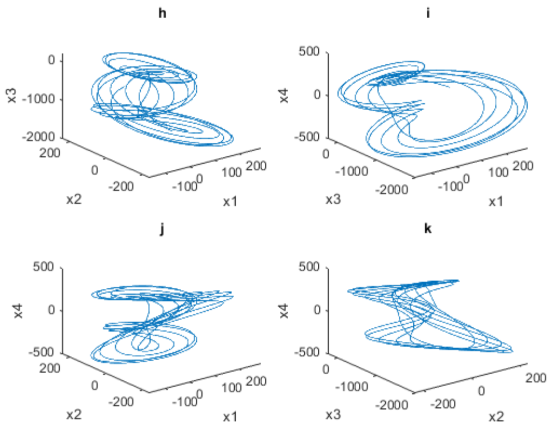
Published By:
Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP)



[Fig.1: Numerical Results of Phase Portraits of (1) using MATLAB. (a) Corresponds to the Phase Portrait with the Transient Phase. (b) Corresponds to the Permanent (Real) Phase of the System]



[Fig.2: Numerical Results of 2D (Dimension) Phase Portraits of the (1) using MATLAB]



[Fig.3: Numerical Results of 3D Phase Portraits of the (1) using MATLAB]

B. Matrix-Based Method to Solve Coupled Differential Equations

Transform (1) into the form as in (9).

$$\frac{dy}{dt} + Ay + u + z \quad \ldots \quad (9)$$

(2) can be written as follows, as in (10):

$$\frac{dy}{dt} = -Ay - u - z \quad \ldots \quad (10)$$

Demonstration of (8) into the Matrix form (x_1, x_2, x_3, x_4 changed into y_1, y_2, y_3, y_4) to make it in the same format (Note: to make y_1^2 and y_3^2 we made changes in order as y_2, y_3, y_4, y_1) as in (11):

$$\frac{dy}{dt} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -a_2 & 0 & -a_1 \\ 0 & 0 & 1 & 0 \\ 0 & -b_1 & 0 & -b_2 \end{bmatrix}}_{A} * \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}}_{Y} + \underbrace{\begin{bmatrix} 0 \\ a_3 \\ 0 \\ b_3 \end{bmatrix}}_{U} + \underbrace{\left(\left(\begin{bmatrix} y_2 \\ y_3 \\ y_4 \\ y_1 \end{bmatrix} * \begin{bmatrix} y_2 \\ y_3 \\ y_4 \\ y_1 \end{bmatrix} \right) * \begin{bmatrix} 0 \\ a_4 \\ 0 \\ b_4 \end{bmatrix} \right)}_{Z} \quad \ldots \quad (11)$$

Determining the parameters A, U and the variable Y and Z in (11), the following assignments can be concluded:

$$A = [1\ 0\ 0\ 0;\ 0\ -a_2\ 0\ -a_1;\ 0\ 0\ 1\ 0;\ 0\ -b_1\ 0\ -b_2];\ Y = [y_2; y_3; y_4; y_1];\ U = [0; a_3; 0; b_3];\ C = [0; a_4; 0; b_4].$$

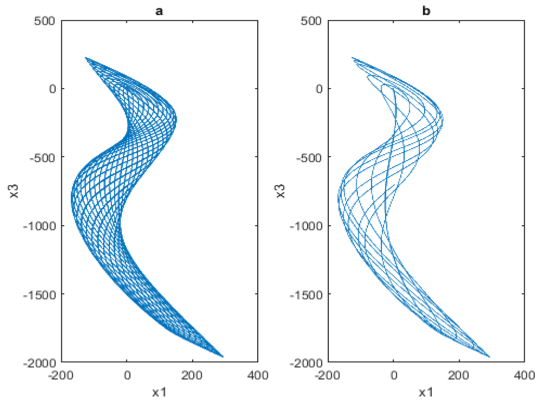
The variable Z can be derived as in (12):

$$Z = (Y.*Y).*C \quad \ldots \quad (12)$$

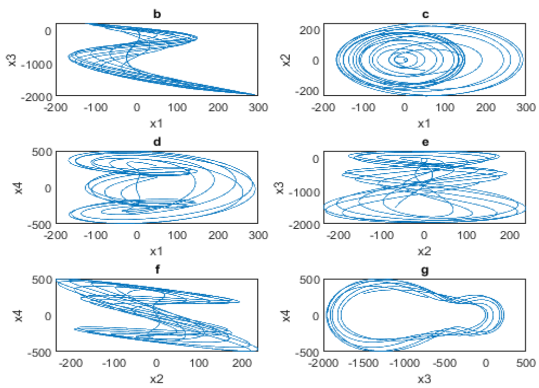
Then, we can obtain that as in (13):

$$\frac{dy}{dt} = A * Y + U + Z \quad \ldots \quad (13)$$

Plot of all solutions and phase portraits of (11), which is demonstrated in Fig. 1.2.1 – Fig. 1.2.3.



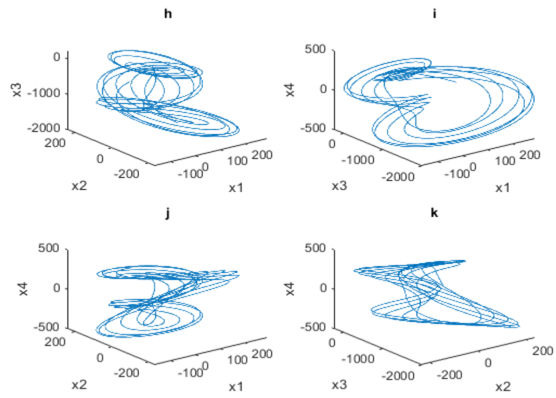
[Fig.4: Numerical Results of Phase Portraits of (11) using MATLAB. (a) Corresponds to the Phase Portrait with the Transient Phase. (b) Corresponds to the Permanent (real) Phase of the System]



[Fig.5: Numerical Results of 2D (Dimension) Phase Portraits of (11) Using MATLAB]

Retrieval Number: 100.1/ijies.H111712080825
DOI: 10.35940/ijies.H1117.12080825
Journal Website: www.ijies.org

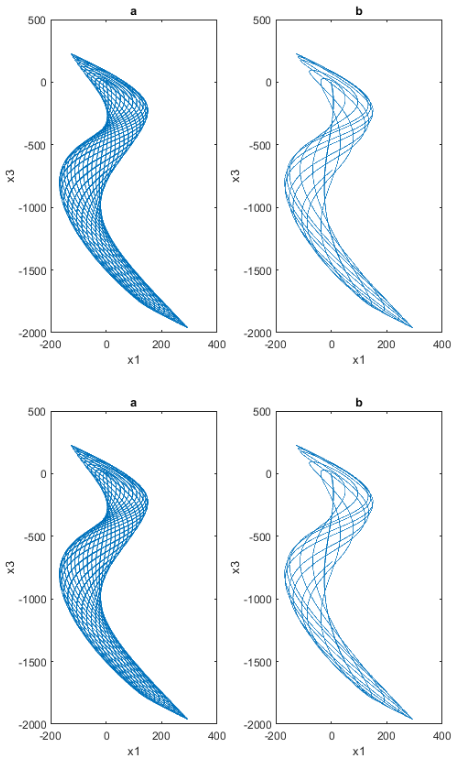
Published By:
Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP)



[Fig.6: Numerical Results of 3D Phase Portraits of (11) using MATLAB]

IV. COMPARATIVE ANALYSIS OF THE DISTINCTIONS WITH NUMERICAL RESULTS

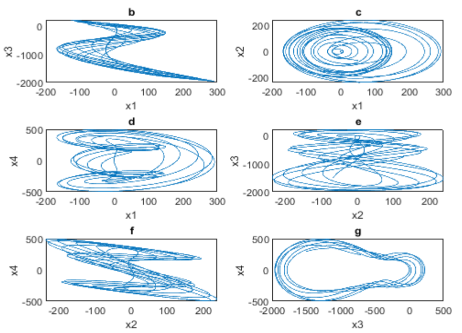
Comparison of all the results from the plots obtained using (1) with those obtained using (11), as demonstrated in Figs. 1.3 – 1.5.



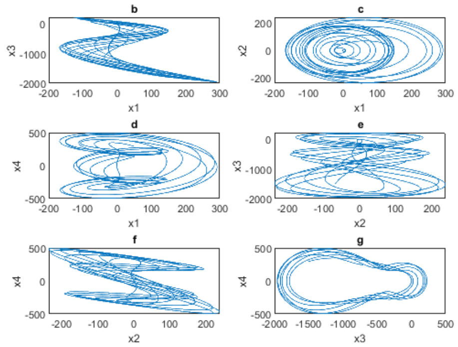
A

B

[Fig.7: Comparison of the same numerical results of phase portraits of (1) and (11). (A) Corresponds to the phase portrait of (1) and (B) with plots produced by (11)]

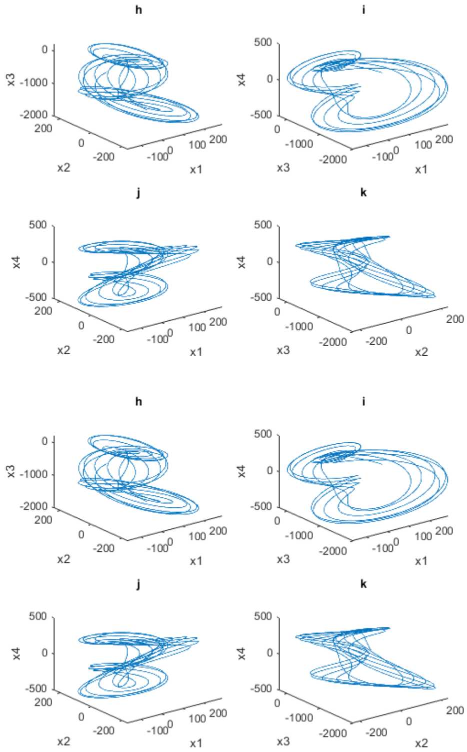


A



B

[Fig.8: Comparison of the Same Numerical Results of 2D (Dimension) Phase Portraits of (1) and (11). (A) Corresponds to the 2D (Dimension) Phase Portraits of (1) and (B) with Plots of (11)]



A

B

[Fig.9: Comparison of the Same Numerical Results of 3D (Dimension) Phase Portraits of (1) and (11). (A) Corresponds to the 3D (Dimension) Phase Portraits of (1) and (B) with Plots (11) in 3D]

Depending on the numerical results obtained by implementing distinct methods, it is essential to emphasise that the corresponding system gives identical phase portraits.

V. CONCLUSION

To summarise, combining numerical methods with matrix-based high-performance solvers offers a powerful approach to addressing the challenges posed by coupled differential equation models. From our investigation, we have demonstrated that this multidisciplinary method is effective in enhancing the computing efficiency, scalability, and accuracy of complex dynamical system simulations. We have been able to overcome the drawbacks of conventional computational techniques and

Retrieval Number: 100.1/ijies.H111712080825
DOI: 10.35940/ijies.H1117.12080825
Journal Website: www.ijies.org

Published By:
Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP)

uncover new perspectives on the behaviour of coupled systems in various areas by leveraging the adaptive learning capabilities of neural networks and the efficiency of matrix-based solvers. Our results demonstrate how this strategy can foster scientific knowledge and creativity across diverse sectors, including engineering, finance, biology, and physics. Subsequent research endeavours should focus on refining algorithms, expanding the range of applications, and validating the approach in various real-world scenarios. Ultimately, the combination of numerical and matrix-based solvers has enormous potential to transform the way we model and evaluate complex systems, thereby promoting scientific research and technological innovation.

DECLARATION STATEMENT

I must verify the accuracy of the following information as the article's author.

- **Conflicts of Interest/ Competing Interests:** Based on my understanding, this article has no conflicts of interest.
- **Funding Support:** This article has not been funded by any organizations or agencies. This independence ensures that the research is conducted with objectivity and without any external influence.
- **Ethical Approval and Consent to Participate:** The content of this article does not necessitate ethical approval or consent to participate with supporting documentation.
- **Data Access Statement and Material Availability:** The adequate resources of this article are publicly accessible.
- **Author's Contributions:** The authorship of this article is contributed solely.

REFERENCES

1. C. S. Varun Bose, R. Udhayakumar, M. Savatović, A. Deiveegan, V. Todorčević, and S. Radenović, "Existence of Mild Solution of the Hilfer Fractional Differential Equations with Infinite Delay on an Infinite Interval," Fractal Fract., vol. 7, no. 10, pp. 1–12, 2023, DOI: https://doi.org/10.3390/fractalfract7100724
2. Venkatachalapathi Uday and Gautam Kumar Rajput, "Differential Equations: The Mathematical Framework for Modelling Dynamic Systems," Int. J. Curr. Res. Tech., vol. 14, no. 4, pp. 50246–50249, 2024, DOI: https://doi.org/10.61359/2024050028
3. C. Szabo, "Complex Systems Modelling and Analysis," Proc. - Winter Simul. Conf., pp. 87–100, 2024, DOI: https://doi.org/10.1109/WSC63780.2024.10838891
4. S. B. Doma, "Applications in Physics and Engineering Part 1 : Second Order Partial Differential Salah Badawi Doma Awad Hamdy El-Sharif Assistant Professor of Mathematics," vol. 10, no. 1, 2021. https://www.researchgate.net/publication/354969264
5. V. Tavakkoli, J. C. Chedjou, and K. Kyamakya, "A novel recurrent neural network-based ultra-fast, robust, and scalable solver for inverting a 'time-varying matrix,'" Sensors (Switzerland), vol. 19, no. 18, pp. 1–20, 2019, DOI: https://doi.org/10.3390/s19184002
6. M. Arabboev et al., "Development of a novel method for image resizing using Artificial Neural Network," in the 14th International Conference on Intelligent Human Computer Interaction, Tashkent, Uzbekistan, 2022, p. 35. DOI: https://dx.doi.org/10.1007/978-3-031-27199-1_53
7. S. Tashmetov, J. C. Chedjou, K. Nosirov, A. Berdiyev, and E. Norov, "Computational Approaches for Solving Complex Differential Equations: Implementation of the Direct Numerical Method with Modelling and Analysis of Bifurcation Diagrams and Lyapunov Exponent," Proc. - Int. Conf. Inf. Sci. Commun. Technol. - Appl. Trends Oppor. ICISCT 2024, pp. 151–155, 2024, DOI: https://doi.org/10.1109/ICISCT64202.2024.10957210
8. E. Norov, S. Tashmetov, K. Nosirov, M. Rakhmatullaeva, A. Yusupov, and J. C. Chedjou, "On the Convergence of an Efficient and Robust Dynamic Neural Network Concept with Application to Solving Travelling Salesman Problems," WSEAS Trans. Circuits Syst., vol. 21, pp. 285–294, 2022, DOI: https://doi.org/10.37394/23201.2022.21.31
9. M. Agop et al., "Toward complex systems dynamics through flow regimes of multifractal fluids," Symmetry (Basel), vol. 13, no. 5, 2021, DOI: https://doi.org/10.3390/sym13050754
10. D. Levy Dan Bree, "HHS Public Access," Physiol. Behav., vol. 176, no. 3, pp. 139–148, 2019, DOI: https://doi.org/10.1016/j.jcrc.2008.02.003.EQUATION-BASED

AUTHOR'S PROFILE

Shakhzod Tashmetov, received his Bachelor's degree in Audiovisual technologies from the Tashkent University of Information Technologies named after Muhammad al-Khwarizmi in 2017, and received his Master's degree in Audiotechnologies from the Tashkent University of Information Technologies named after Muhammad al-Khwarizmi in 2019 is now pursuing to hold a Ph.D. dissertation defense on the Systems and systems of radio engineering, radio navigation and television, Mobile, fiber-optic communication systems specialty by the order of the Higher Attestation Commission under the Cabinet of Ministers of the Republic of Uzbekistan. He worked as a Trainee-Researcher **at** Tashkent University of Information Technologies, named after Muhammad al-Khwarizmi (2019–2020), and is now an Assistant Teacher at this university. His research topics mainly include: image processing, deep learning, differential equations, and nonlinear dynamical systems.

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of the Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP)/ journal and/or the editor(s). The Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP) and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions, or products referred to in the content.